Information Retrieval
WS 2016 / 2017

Lecture 3, Tuesday November 8th, 2016
(Efficient List Intersection)

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Overview of this lecture

- Organizational
  - Your experiences with ES 2
  - About the tutorials

- Contents
  - List Intersection
  - Non-algorithmic improvements
  - Algorithmic improvements

Exercise Sheet 3: implement list intersection and make it as fast as possible on a small benchmark we have prepared
Experiences with ES2  1/3

Summary / excerpts

- Time-extensive exercise sheet, but again mostly due to lack of programming experience or practice
- Some mistakes in the TIP file, but quickly fixed when pointed out on the forum
  
  Please always watch the forum for updates!

- Some problems with the floats (1.885) in the test cases
- Some bugs not found by test cases
- Lecture recording helped a lot
- One does not program well with: a cold, lack of sleep, lack of concentration, starting late, ...
Experiences with ES2  2/3

Results

- Small differences in the implementation can make a significant difference in the results
- Variation of the BM25 parameters make some queries better, but don't affect others or make them worse
- Boosting (popular documents, full match) helped a bit
- Removing frequent words ("stop words") helped a bit
- Best results: $P@3 \approx 60\%$, $P@R \approx 40\%$, MAP $\approx 40\%$
- Bottom line: tuning a ranking algorithm is super important (for result quality) but also super hard

In particular, it is very hard to understand / predict the effect of changes in the parameters / implementation
Test Cases

- A working program written by yourself is a confirmation that you have really understood the stuff from the lecture

- However: this only holds true if the program does what it should do and not just "something"

  Experience from (many) previous courses: a significant fraction of students submit code that technically "works" but is otherwise wrong or even completely wrong

  E.g. for ES2, a completely wrong ranking

- This is why we make the effort to provide test cases for you and require from you that you implement them

  This has actually been a frequent request in previous years
Online vs. personal

- So far, the tutorials are completely online:
  You submit online, you get feedback online, and questions and answers are online

- This is great because it gives you a lot of flexibility (you can work and ask any time you prefer)

- However, some people prefer a personal meeting, at least from time to time ... hence we offer:
  You can meet with your tutor anytime, just ask him/her
  There will be a personal tutorial every four weeks or so

**QUESTION: when would be a good time for you?**
Recap and motivation for today

- In Lecture 1, we have intersected the inverted lists.
- In Lecture 2, we have merged the inverted lists.
- For efficiency reasons, many search engines only return results which contain all the query words.

Apache's Lucene, the most widely used open-source search engine, supports intersect (AND) and merge (OR).

In most applications, intersect is used by default.

- Today we will focus on **efficiency** and therefore on list intersection.
Time measurement

- Trickier than it may seem at first, because there can be significant variation between runs, for example due to:
  
  Other jobs running on your machine
  The Java garbage collector running unpredictably
  Data is partly in disk cache / L1-cache / TLB cache

- Therefore, always repeat your time measurements, and take the average over all these

For ES3, **repeat 5 times** for each measurement

Note: repetition itself can also distort the truth because of caching effects ... but not an issue for us today
Time measurement in **Java**

- For **milli**second resolution

  ```java
  long time1 = System.currentTimeMillis();
  // whatever code you want to time
  long time2 = System.currentTimeMillis();
  long millis = time2 – time1;
  ```

- For **micro**second resolution

  ```java
  long time1 = System.nanoTime();
  // whatever code you want to time
  long time2 = System.nanoTime();
  long micros = (time2 – time1) / 1000;
  ```
Time measurement in C++

- For millisecond resolution (C-Style)  
  ```c
  #include <time.h>
  clock_t time1 = clock();
  // whatever code you want to time
  clock_t time2 = clock();
  size_t millis = 1000 * (time2 - time1) / CLOCKS_PER_SEC;
  ```

- For microsecond resolution (C++11)  
  ```cpp
  #include <chrono>
  auto time1 = std::chrono::high_resolution_clock::now();
  // whatever code you want to time
  auto time2 = std::chrono::high_resolution_clock::now();
  size_t micros = std::chrono::duration_cast<std::chrono::microseconds>(...).count();
  ```
Non-algorithmic improvements 1/4

- **Motivation**
  - Implementation details can have a great impact on performance (even with the same underlying algorithm)
  - Let us implement the basic "zipper" algorithm for list intersection from Lecture 1 and look at a few variations
  - We make a part of the code (reading from file and the basic algorithm) available to you in both Java and C++
    This should make ES3 easier / less work for you
  - During the lecture, I will implement in Java today

Note that using **Python** makes little sense when studying efficiency issues: the overhead of its internal data types (i.p. Python's lists/arrays) weighs too heavy
Non-algorithmic improvements 2/4

- **Native arrays**
  - **Java**: `ArrayList` much worse than native `[]` array
    
    Elements of an ArrayList cannot be basic data types (e.g. `int`), but have to be objects (e.g. `Integer`)
    
    **This causes inefficient byte code / machine code**
  
  - **C++**: `std::vector` is as good as `[]` with option `-O3`
    
    Elements of an std::vector can be basic data types as well as objects
    
    **Due to C++'s templating mechanism, machine code for std::vector<int> is almost the same as for int[]**
Predictable branches

- Branches = all conditional parts in your code
  
  In particular, **if ... then ... else** parts

- Modern processors do pipelining = speculative execution of future instructions before the current ones are done

- For conditional parts they have to guess the outcome

- So good to **minimize** amount of conditional parts and/or improve the predictability of conditionals

  A conditional has good predictability if it evaluates to the same Boolean value most of the time
Non-algorithmic improvements  4/4

- Sentinels
  - Special elements to avoid testing for index out of bound
    
    Less code + further reduction in number of branches
  
  - For list intersection: id $\infty$ at the end of both lists
    
    For Java, take:  `Integer.MAX_VALUE`
    
    For C++, take:  `std::numeric_limits<int>::max()`
Algorithmic improvements 1/8

■ Preliminaries

– We have two lists, which we want to intersect

– Let $A$ be the smaller list, with $k$ elements

– Let $B$ be the longer list, with $n$ elements

List intersection is commutative, so we can always assume that the first list is $A$, and the second is $B$

– Recall that both lists are sorted ... this is crucial for the basic algorithm and all the algorithms in the following
Algorithmic improvements 2/8

- Binary search in the longer list
  - Search each element from $A$ in $B$, using binary search
  - This has time complexity $\Theta(k \cdot \log n)$

Good for small $k$ ... but for $k = \Theta(n)$ this is $\Theta(n \cdot \log n)$, and hence slower than the "zipper"-style linear intersect
Algorithmic improvements

- Binary search in remainder of longer list
  - Time complexity in the best case $\Theta(k + \log n)$
  - First element from $A$ towards the end of list $B$
  - Time complexity in the worst case $\Theta(k \cdot \log n)$
  - All elements of $A$ at the beginning of list $B$
  - Time complexity in the "typical" case $\Theta(k \cdot \log n)$
  - Elements of $A$ "evenly distributed" over list $B$

A ← B
A[0] ← A[0] + B

because
\[
\log \frac{m}{2} \approx \log \frac{m}{\epsilon} \\
\text{as } \epsilon \ll m
\]
Galloping search

- **Goal:** When elements $A[i]$ and $A[i+1]$ are located at positions $j_1$ and $j_2$ in $B$, then, with $d := j_2 - j_1$ ("gap"): spend only time $\Theta(\log d)$ to locate element $A[i+1]$.

- **Idea:** First do an exponential search, to get an upper bound on the range, then a binary search as before.
Algorithmic improvements  5/8

- **Galloping search, time complexity**
  - Let $j_1, \ldots, j_k$ the positions of the elements of $A$ in $B$
  - Let $d_i = j_i - j_{i-1}$ for $i > 1$ and $d_1 = 1$ (the "gaps")
  - Note that $\sum d_i \leq n =$ the number of elements in $B$
  - Then the time complexity is $O(\sum \log d_i)$
    - Not a nice formula, so let's find the maximum value, independent of the particular $d_1, \ldots, d_k$
    - Lemma: $\sum \log d_i$ is maximized when all $d_i = n/k$
    - Galloping search therefore takes time $O(k \cdot \log (1 + n/k))$
      - This is always $O(n)$ and hence never worse than "Zipper"

**VERIFY YOURSELVES!**
Proof of Lemma … max $\sum_i \ln d_i$ under constraint $\sum_i d_i \leq n$

- This is an instance of **Lagrangian optimization**:

1. Write constraint as equation: $\sum_i d_i - n' = 0$ ... $n' < n$

2. Define $L(d_1, ..., d_k, \lambda) = \sum_i \ln d_i + \lambda \cdot (\sum_i d_i - n')$

3. Set partial derivatives = 0 to find all local optima and check the objective function at the borders

$$\frac{\partial L}{\partial d_i} = \frac{1}{d_i} - \lambda = 0 \quad \Rightarrow \quad d_i = \frac{1}{\lambda}$$

\[\text{are all equal} \quad (*)\]

$$\frac{\partial^2 L}{\partial d_i^2} = -\frac{1}{d_i^2} < 0 \quad \Rightarrow \text{we have a MAX at} \quad d_i = \frac{1}{\lambda}.$$

\[\text{(*) } \Rightarrow \ d_i = \frac{m^i}{2^i} \quad \Rightarrow \ \sum_{i=1}^{\infty} \ln d_i \leq \sum_{i=1}^{\infty} \frac{m^i}{2^i} \leq 2 \cdot \log \frac{M}{2e} \]
Comparison-based lower bound

- Recall the lower-bound for comparison-based sorting
  
  There are \( n! \) possible outputs, we have to differentiate between all of them, and only two choices per step.
  
  Hence \( \#\text{steps required} \geq \log_2(n!) = \Omega(n \cdot \log n) \)

- We can use a similar argument for intersection / union:
  
  There are \( \binom{n+k}{k} \) ways how the \( k \) elements from \( A \) can be placed within the \( n \) elements from \( B \), ...
  
  Hence \( \#\text{steps required} \geq \log_2 \left( \frac{n}{k} \right)^k = k \cdot \log_2 \left( \frac{n}{k} \right) \)

Galloping search is hence asymptotically optimal
Skip Pointers

- **Idea**: potentially skip large parts of longer list B
- Skip pointer = special element in list B with a value \( x \) and the index \( j \) of the first element in B with \( B[j] \geq x \)

When intersecting, follow pointer if current \( A[i] \geq x \)

Placement of skip pointers is heuristic ... for ES3 you can investigate good placements experimentally

- Advantage: **very simple** to implement

In particular, simpler than galloping search and thus often more effective in practice, even if not "optimal"
References

- **Textbook**
  
  *Section 2.3: Faster intersection with skip pointers*

- **Literature**
  
  *A simple algorithm for merging two linearly ordered sets*
  
  F.K. Hwang and S. Lin  

  *A fast set intersection algorithm for sorted sequences*
  
  R. Baeza-Yates  
  CPM, LNCS 3109, 31–39, 2004

- **Wikipedia**
  