① Max 3-SAT

↳ if # clauses $a \leq 7 \Rightarrow$ all clauses can be satisfied

---

$x_1 \vee x_2 \vee x_3$

min: 8 assignments for variables

→ 1 clause only invalidates one assignment

7 clauses only invalidate 7 assignments

→ at least 1 assignment still satisfies the formula

---

our LV produces solution L

$|L| \geq \frac{7}{8}a \qquad a \leq 7 \Rightarrow \frac{7}{8}a > a-1$

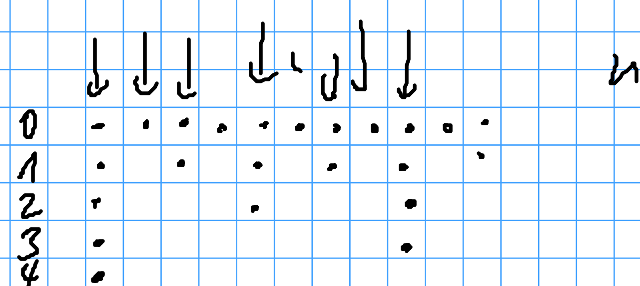
$7a > 8a - 8$

$8 > a$

$\Rightarrow |L| \in \mathbb{N} \rightarrow |L| = a$

---

② Skip Lists



Det. Skip Lists

- Space cons. of list i: $\frac{n}{2^i}$

$$\sum_{i=0}^{\log n} \frac{n}{2^i} = n \sum_{i=0}^{\log n} \left(\frac{1}{2}\right)^i \le n \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i \le n \cdot 2 \in O(n)$$

q^i

Rand. Skip Lists

X_i – counts the lists in that element i appears

X – total space consumption

$$E(X) = \sum_{i=1}^{n} E(X_i) = \sum_{i=1}^{n} 2 = 2n$$

$|U| = n$

(3) Hashing

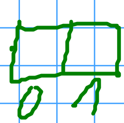
$H := \{ f : U \to \{0, \dots, m-1\} \}$

to show: $\left|\{ h \in H \mid h(u) = h(v) \}\right| \le \frac{|H|}{m}$

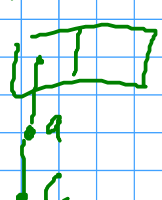
$U = \{a, b, c, d\}$

↳ a, c

hash table of size 2



↳ fix position of a, c as 0



$|H| = m^n$

for fixed position of u and v in the hash table there are m^{n-2} function in H that realize this collision

in total $m \cdot m^{n-2}$ many function in H, which produce a collision for fixed $u, v \in U$

$$m^{n-1} \le \frac{m^n}{m} = m^{n-1}$$

ac bd | ac d b | ac b d | ac bd

4
4 } 8

$2^4 = 16$
many functions

$m = 2$
$n = 4$

$2^{4-1} = 2^3 = 8$

---

④ Fingerprinting

H is universal

chance of collision ~~that~~ $\frac{1}{m}$

k bits: 2^k entries in hash table

$\frac{1}{2^k}$ (chance of collision for the scripts)

$$\underbrace{\frac{1}{2^k} \cdot \frac{1}{2^k} \cdot \ldots}_{c \text{ times}}$$

$$\left(\frac{1}{2^k}\right)^c = \frac{1}{2^{k \cdot c}}$$