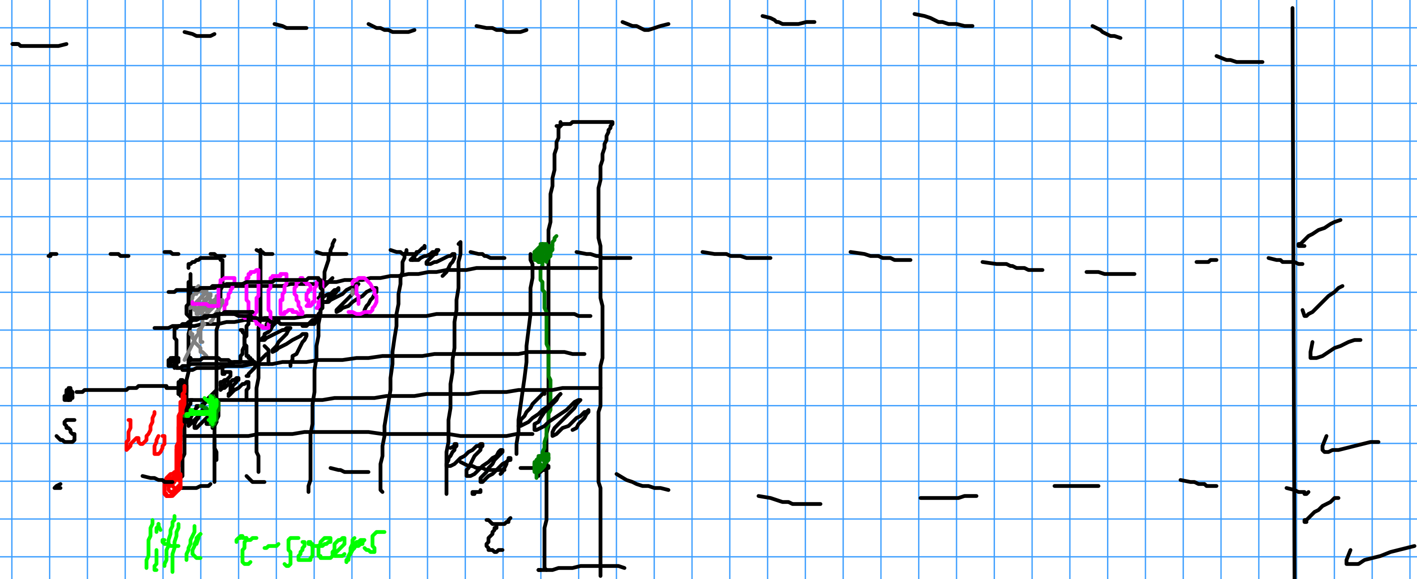



$O(w_i \, n^{\frac{4}{9}} \log n)$

$O(\sqrt{n})$ det.

$O(n^{\frac{4}{9}} \log n)$ rand.

t

4. big τ-sweeps

↳ motion in the correct stripe of the desired field to the right

- For each of the at most $32\sqrt{\tau}$ substages and for each of the at most $2\sqrt{\tau}+1$ stripes, the robot makes at most $\sqrt{L}$ big $\tau$-sweeps before a stripe is declared useless. Each sweep costs $2h = \frac{2W_i}{\sqrt{L}}$ for motion that is upwards / downwards (not around obstacles). Thus, for each phase the cost is $O(W_i \sqrt{L})$. For the whole stage this is $O(W_i \cdot L)$.

When colliding with an obstacle, we make a $\tau_2$-circumvention if possible.

Region in which we run $\tau_2$-sweeps are either above or below fields of strips that we mark as non-active.

So the number of regions

is at most $a_j - a_{j+1}$.

The width of a field

is $\frac{\alpha}{a_j}$. So in subphase

$j$ of a phase, for this portion

of work we pay

$$O\left((a_j - a_{j+1}) \cdot \left(\frac{\alpha}{a_j}\right) \cdot \frac{w_i}{L}\right)$$

Since $\tau = \frac{w_i}{L}$. In a

phase this sums to $O\left(w_i \frac{\alpha}{L} \log L\right)$,

and this is at most

$O\left(w_i \frac{\alpha}{\sqrt{L}} \log L\right)$ in all

$32\sqrt{L}$ phases in a stage.

Plugging in our values of $\alpha$ and

$L$ yields a total cost of

$O(w_i n^{\frac{1}{4}} \log n)$.

Adding together the four costs yields a total cost of

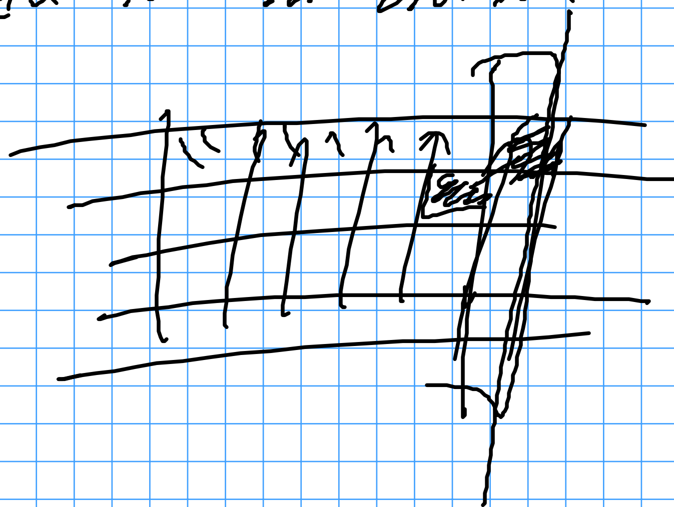
$$O(W_i\, n^{4/3} \log n) \quad \text{for stage } i.$$

c) Remains to show that the probability to reach the target wall is large enough if $W_i$ is $> W$ (optimal).

IDEA: declare an interval $(x_1, x_2)$ as 'good' if progress made in that interval at stage $i$ is at least

$$\frac{1}{2} W_i \sqrt{\epsilon}.$$

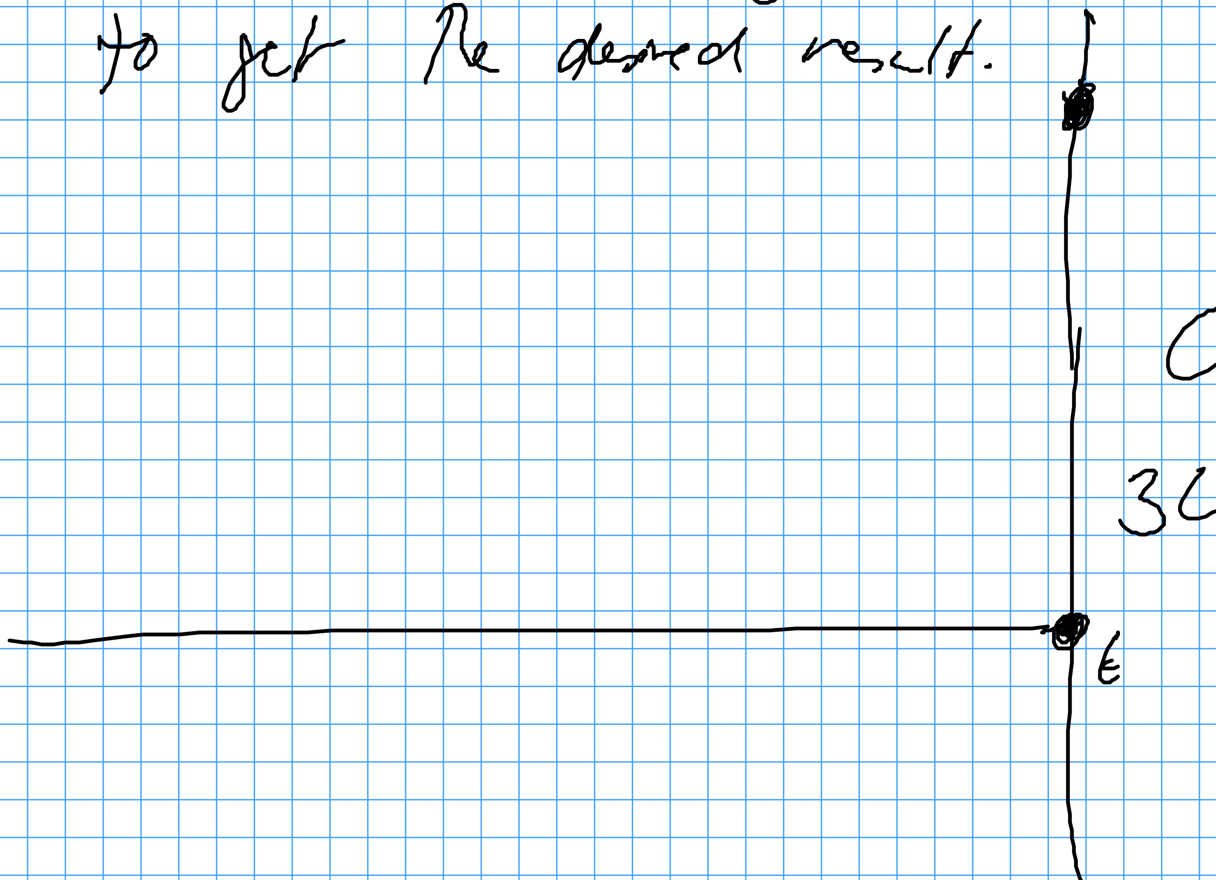
Reasons for not reaching the target wall: Shifts are chosen in a way that movement to the target is blocked or that shifts were declared useless and so the desired field is not visited.



Proof sketch: bound the probability of failure (i.e. the optimal path has to leave the strips considered in stage i) by $\frac{3}{4}$.

The only way a stage can fail to reach the wall is that there are fewer than $2\sqrt{L}$ good phases of the $32\sqrt{L}$ stages.

$\Rightarrow$ use a Chernoff bound to get the prob. of a stage to have that many good plays to get the desired result.




## Online Bidding

Definition: A good has a value of $u$. A player submits a sequence of bids until one is equal or greater to $u$. The player has to pay the sum of all bids in the end.

$d_i$ – $i^{th}$ bid

Competitive ratio $\frac{\sum d_i}{u}$

What values to use for $d_i$ in order to get a good competitive ratio?

$d_i = 2^i$

$d_{last} \leq 2u$

$1+2+4+8+\ldots + 2u$

$$\sum_{i=1}^{\log(u)+1} 2^i = 2^{\log u + 2} = 4u$$

---

Idea for randomization to get a comp. ratio of $e \approx 2.7\ldots$:

choose a bid sequence such that $\frac{d_{last}}{u}$ is in expectation less than the worst case $\frac{d_{last}}{d_{last-1}}$.

Let $d_i = \alpha e^i$ where $\alpha = e^X$

with $X$ randomly distributed

in $[0,1)$. The first term of

the product is

$$\frac{\sum d_i}{d_u} \sim \sum_{i \geq 0} e^{-i} = \frac{1}{1-\frac{1}{e}}$$

$\frac{d_{last}}{u}$ is distributed as $e^X$ with

$X$ uniform in $[0,1)$. The

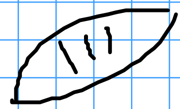
expected value of the ratio is

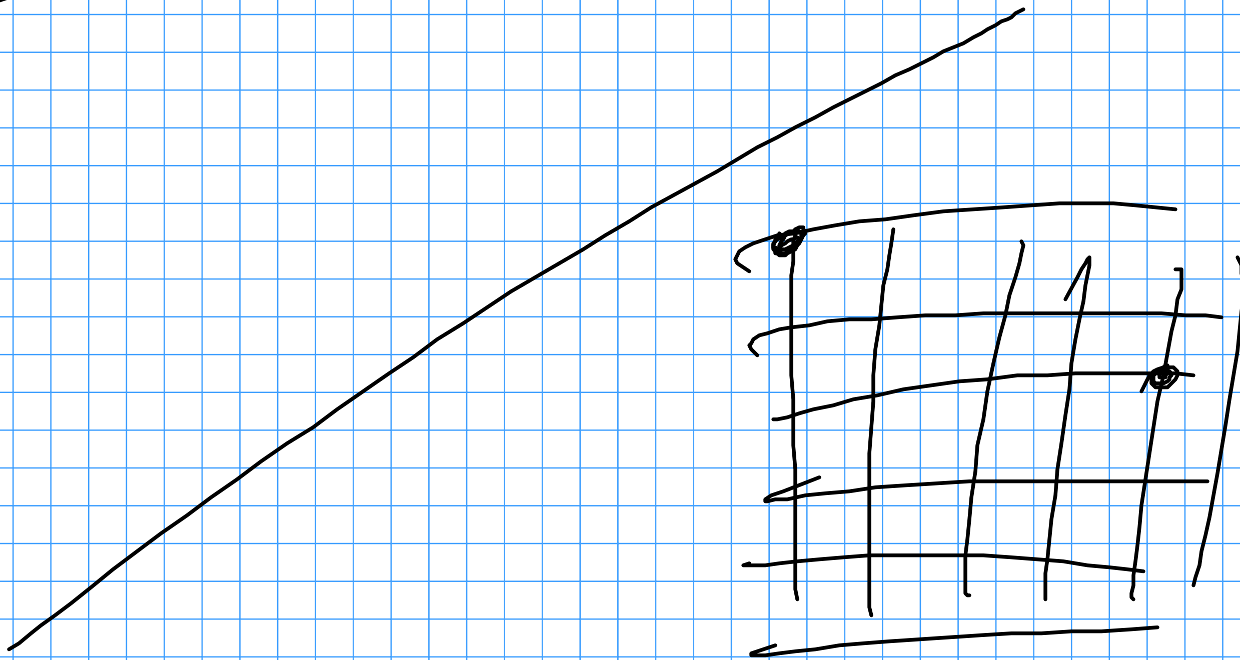
$\int_0^1 e^X dX = e-1$. So the

alg. is $e$-competitive.

(optimal, because rand. lower

bound of $e$ exists)





RIAS

4th August

25th August

Doodle Homepage

$$a_1 \geq a_2 \geq \dots \geq a_{l-1}$$

$$a_l = 0$$

$$\sum_{j=1}^{l-1} \underbrace{\frac{(a_j - a_{j+1})}{a_j}}_{\leq 1} \leq 1 + \ln a_1$$

$\leq a_j$

$$\left(1 - \frac{1}{n}\right)^n = e$$

$\leq 1$

$f(e) \leq 1 + x$