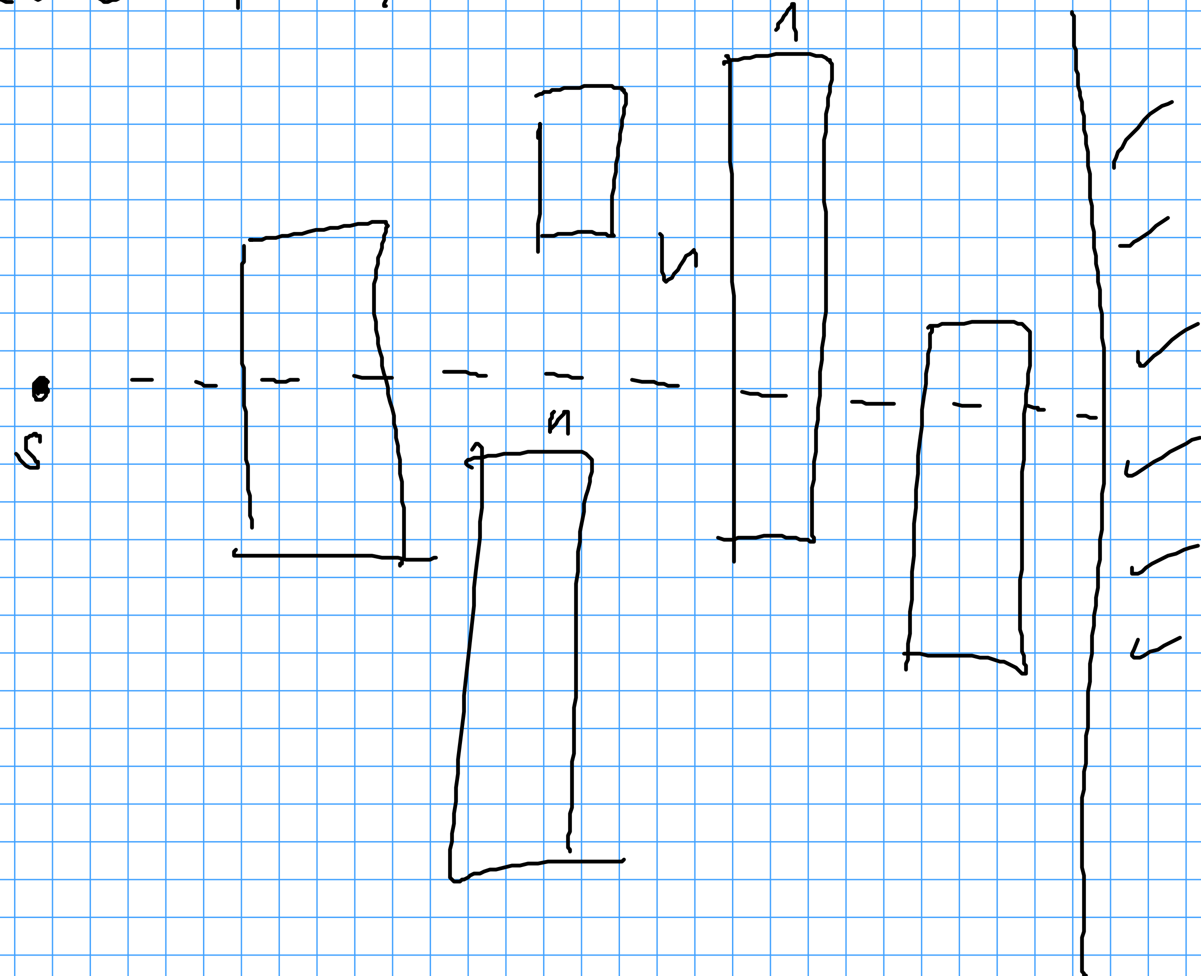
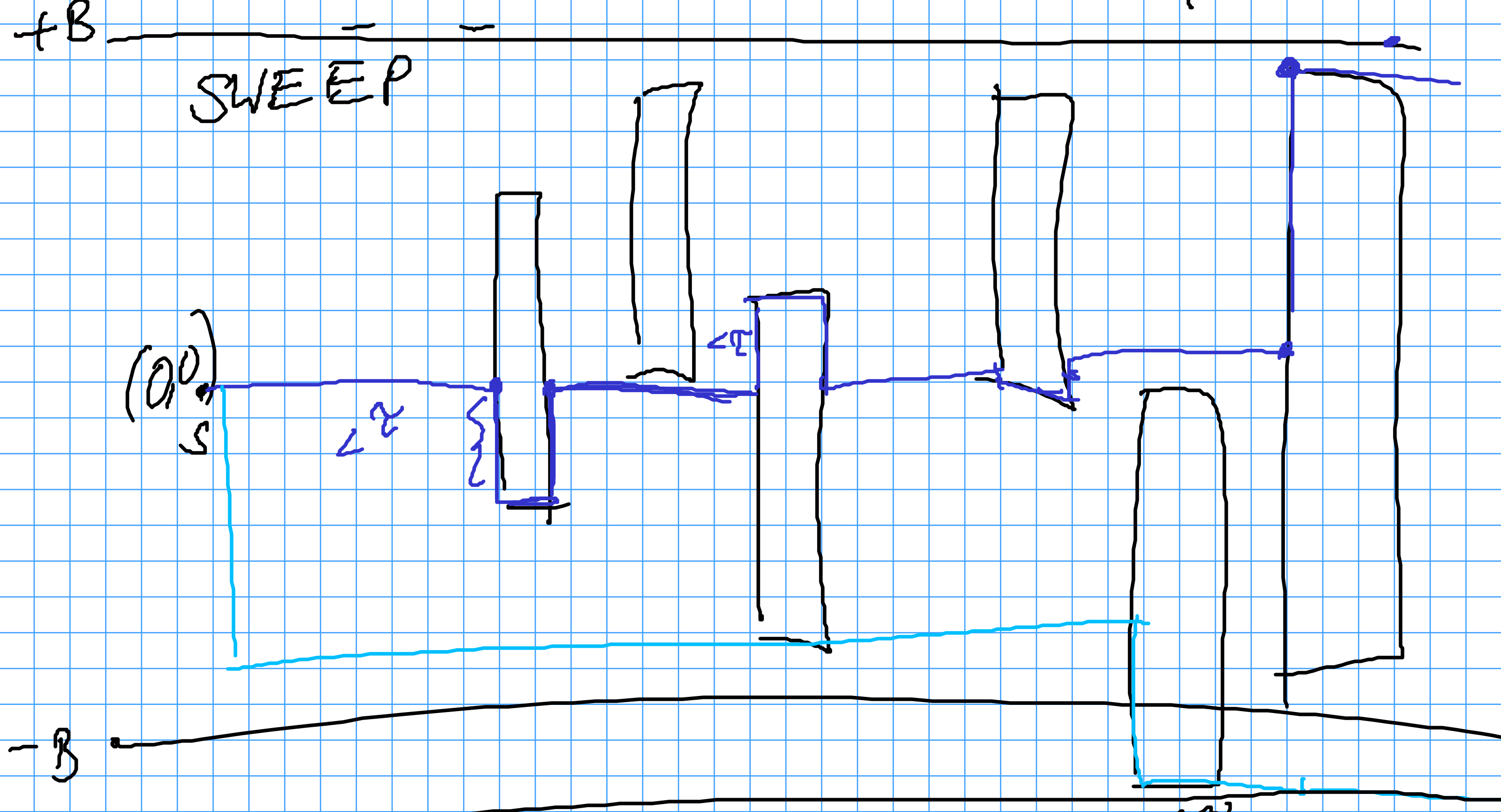
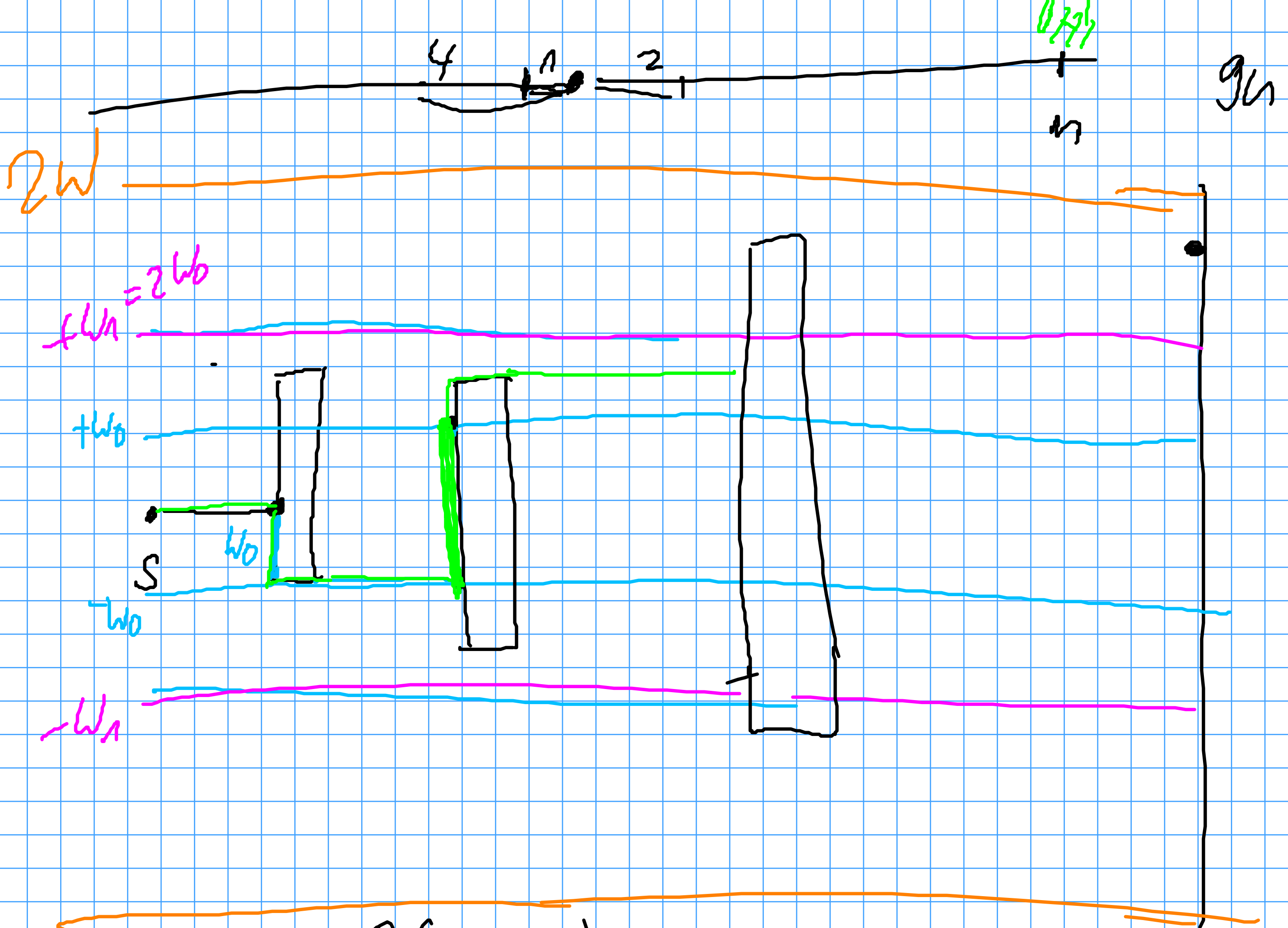
ROBOT NAVIGATION t





W – vertical opt. distance



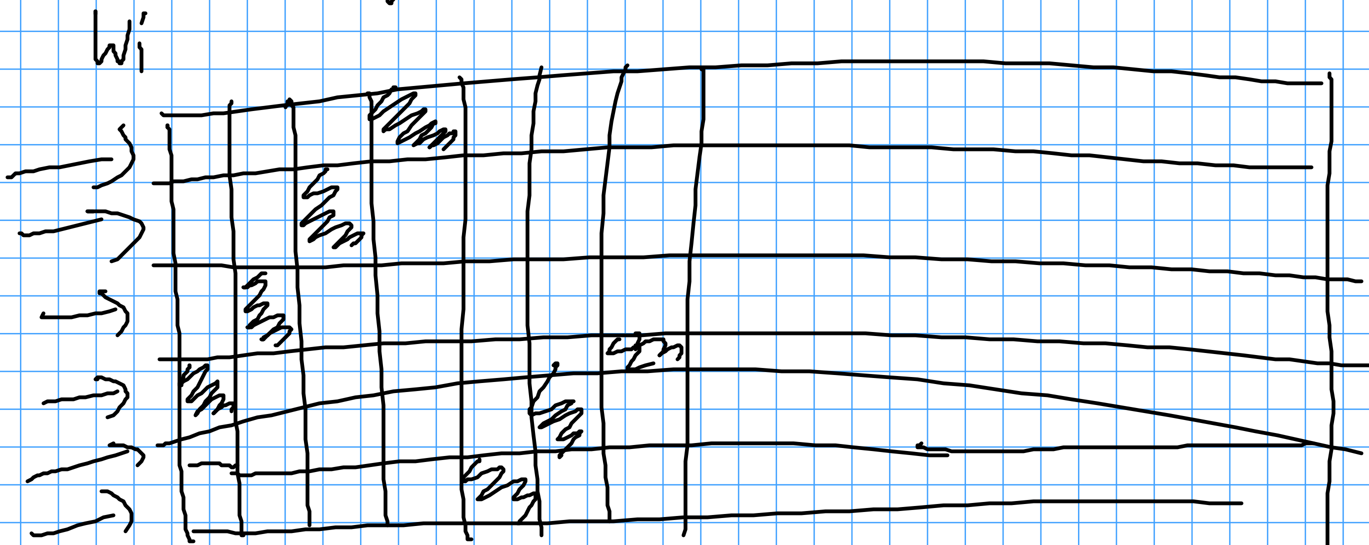
$O(W\sqrt{n})$

$O(\sqrt{n})$ - competitive

$\sqrt{n}$ - lower bound

Randomized Approach: $O(n^{\frac{4}{9}} \log n)$
competitive factor

Stage i



We will describe an algorithm with the following properties:

1. The cost in Stage i is $O(W_i\, n^{\frac{4}{5}} \log n)$

2. For n large enough, for each i with $W_i > W$, the prob. that the robot reaches the wall is $\geq \frac{3}{4}$.

<u>Theorem</u> This alg. is $O(n^{\frac{4}{5}} \log n)$ competitive

Proof: Let j be the integer such that $2^j \cdot w_0 \le W < 2^{j+1} w_0$.

The expected cost of the strategy is

$$E(X) = O\left(\sum_{i=1}^{j-1} w_i \, n^{\frac{4}{9}} \log n + \sum_{i=j}^{\infty} \left(\frac{1}{4}\right)^{i-j} w_i \, n^{\frac{4}{9}} \log n\right)$$

$$= O\left(n^{\frac{4}{9}} \log n \left(\underbrace{\sum_{i=1}^{j-1} 2^i w_0}_{O(2^j w_0) \in O(W)} + \underbrace{\sum_{i=j}^{\infty} \left(\frac{1}{4}\right)^{i-j} 2^i w_0}_{\left(\frac{1}{2}\right)^i w_0 \le 2 w_0 \in O(W)} \right)\right)$$

$$= O\left(n^{\frac{4}{9}} \log n \cdot \left(W + W\right)\right)$$

$$= O\left(W \, n^{\frac{4}{9}} \log n\right)$$

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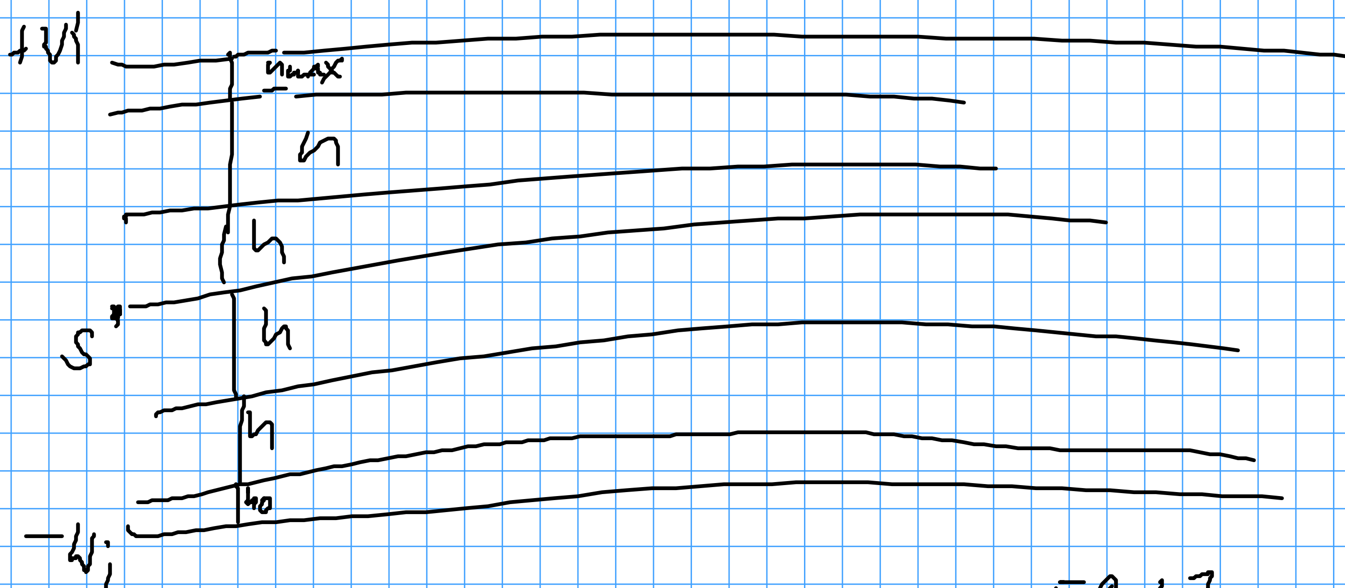
Details of a Stage

- subdivide corridor in stage i

$([-W_i, +W_i])$ in stripes of height h, with $h = \frac{W_i}{n^{\frac{2}{3}}} = \frac{W_i}{\sqrt{C}}$



- choose random value in $[0, h]$, call it h_0
- corridor stripes are then

$[-W_i, -W_i + h_0]$

$[W_i + h_0, -W_i + h_0 + h]$

$\vdots$

last corridor is bounded by $+W_i$

$\rightarrow$ # stripes: $2\left\lceil \frac{W_i}{h} \right\rceil + 1$

stripe labels

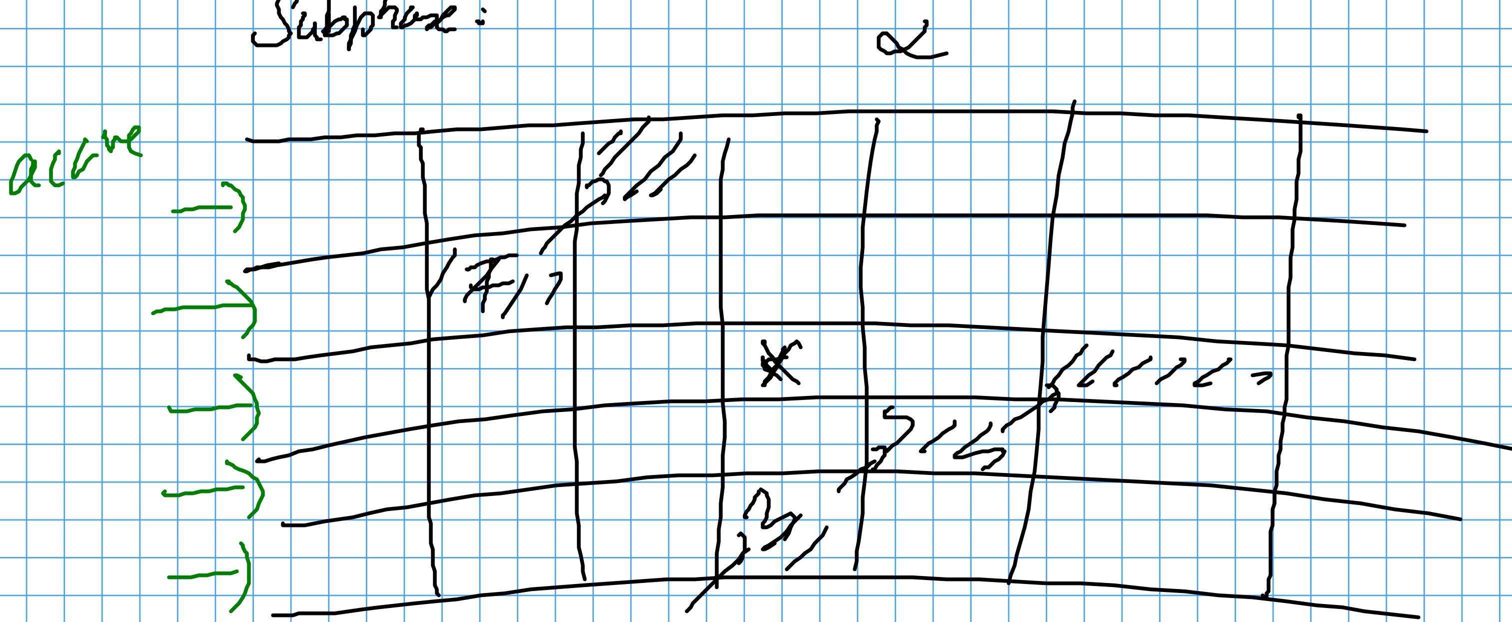
- active
- inactive
- useless

Phase: all stripes are active, perform subphases until target wall is reached or all stripes are non-active (in active or useless)

↳ once all of the stripes are non-active after performing sufficiently many subphases, we start a new phase

$$\alpha = n^{\frac{2}{3}}$$

Subphase:

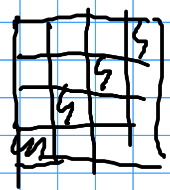


# of active stripes at the beginning of subphase j is a_j

↳ divide the stripes into boxes of equal width $\frac{a}{a_j}$

• choose one active stripe u.a.r., then

construct a 'desired' diagonal.



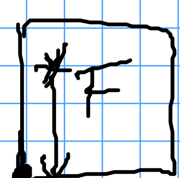
Go always to the upper left field, until there is no active stripe left above, then use a wrap-around to start at the bottom and proceed until every stripe is covered

- How should the robot find its way to the next desired field?

Three cases

(1) robot is in the desired field

(2) inside the stripe containing the field but left of it



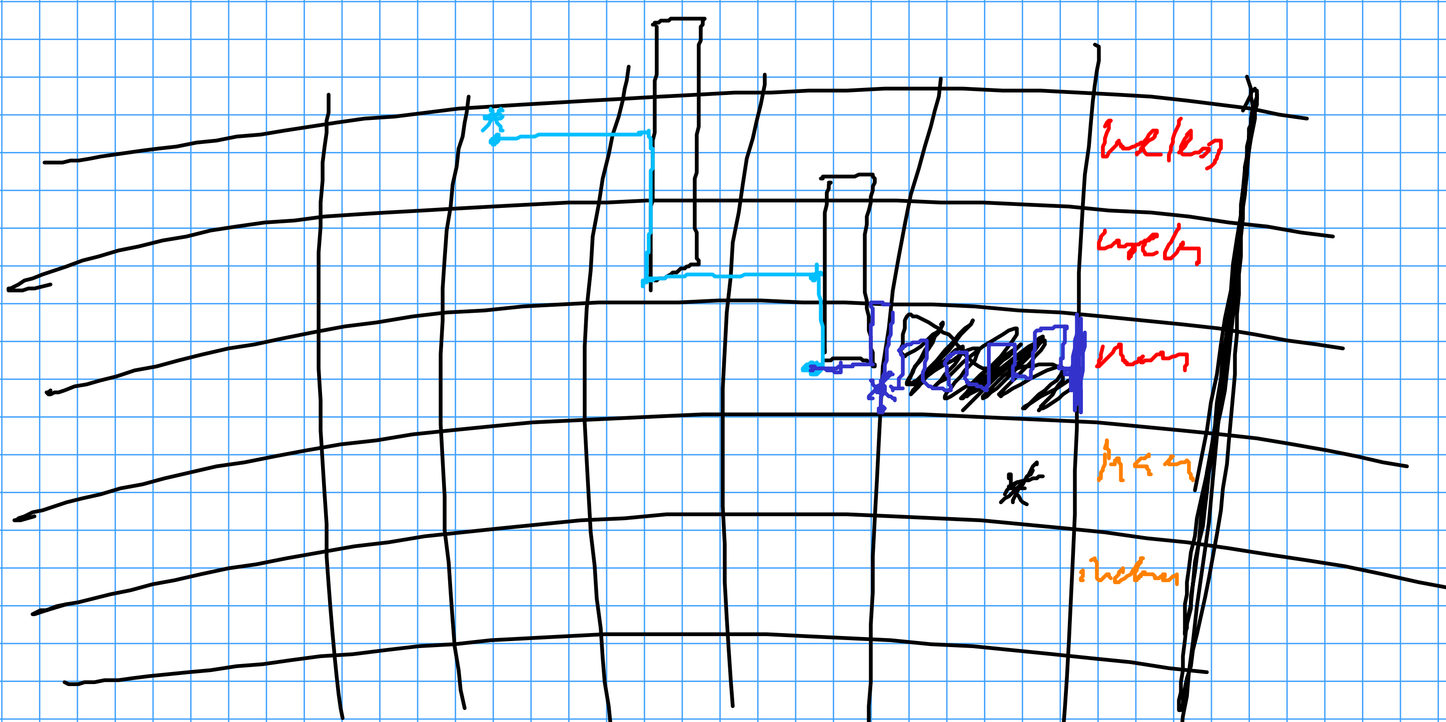
(3) attempting to move to the correct stripe

(1) go greedily to the bottom of the field → perform sweeps with

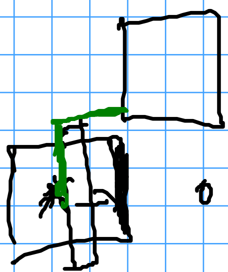
$r_1 = r \cdot \frac{W_i}{L}$ $\quad r = n^{-\frac{1}{5}}$ $\quad L = n^{\frac{4}{5}}$ small sweeps

(2) move towards the field by using sweeps with parameter $\tau_2 = \frac{w_i}{2}$ big sweeps

(3) go greedily towards the correct stripe: go right as long as possible, if an obstacle is hit, go up or down whatever brings you closer to the correct stripe

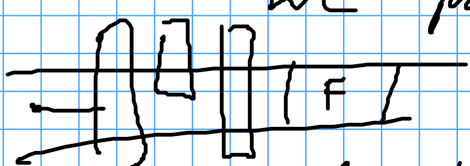


If it happens that the robot is X-coordinate passe the right edge of the desired field before its stripe is reached, then update the desired field to the next one on the 'diagonal'.



- If the robot performs in the desired field $\left\lceil\left(\sqrt{\frac{n}{r}} \cdot \frac{\sqrt{2}}{a_j}\right)\right\rceil$ full small sweeps, we declare the stripe inactive.

After this number of sweeps or after reaching the right edge of the field, we proceed to the next desired one.



- If the robot is in mode (2) then he stops after at most $\sqrt{2} = n^{\frac{2}{3}}$ many big sweeps (counting over the whole phase)

  ↳ if they are used up, declare the stripe useless

By the property of the sweep, a stripe is declared useless only if the cost of any path lying entirely in this stripe is at least $\frac{w_i}{\sqrt{2}} = h$.

Final case: robot has no more desired fields (because whole columns only consist of useless or inactive fields)

'fill-up case'

↳ still need to get to the right edge of the box with width α

⟹ greedily move upwards (with wrap around) to the next non-useless stripes and repeat. If all stripes are useless, then we start a new phase.

Exercise Let $a_1 \geq a_2 \geq \dots \geq a_{\ell-1} > 0$ be integers. Define $a_\ell = 0$.
Then $\sum_{j=1}^{\ell-1} \frac{(a_j - a_{j+1})}{a_j} \leq 1 + \ln a_1$.

Lemma: The cost of the alg. at stage i is $O(w_i \, n^{\frac{4}{3}} \log n)$.

Proof: We analyze the cost of the alg. along y-direction.
We partition it into several parts.

1. going to the correct field costs $O(w_i)$ for each subphase. (Including fill-up in the end).

All subphases cover a distance d in positive x-direction (except maybe the last one)

$\Leftarrow$ at most $\frac{n}{\alpha}$ subphases

of full lengths

$\Rightarrow$ at most $32\sqrt{L}$ phases in a stage $\Rightarrow$ clearly in one stage the robot can cover at most distance n along the x direction, therefore the total of these costs over all subphases in the stage is $O\left(w_i\left(\frac{n}{\alpha}+\sqrt{L}\right)\right)$

$\alpha = n^{-\frac{1}{9}}$ $L = n^{\frac{4}{9}}$ exercise

$\Rightarrow O\left(w_i\, n^{\frac{4}{9}} \log n\right)$

2. now consider small sweeps inside fields in the case that we do <u>not</u> mark the stripe inactive

Denote by a_j the number of stripes active at the beginning of subphase j. Each field has a width of $\frac{\alpha}{a_j}$.

The cost in the field is in $O\left(\left(\frac{\alpha}{a_j}\right) \cdot \frac{rW_i}{L} + \left\lceil\frac{\sqrt{L}}{a_j r}\right\rceil \frac{W_i}{\sqrt{L}}\right)$

↳ first term is the cost of going around at most $\frac{\alpha}{a_j}$ many obstacles which are hit within T_n of the nearest corner ($T_n = \frac{rW_i}{L}$)

↳ second term is the cost of doing at most $\left\lceil\frac{\sqrt{L}}{a_j r}\right\rceil$ full sweeps in the corridor of length $h = \frac{W_i}{\sqrt{L}}$

↳ This gives us a cost of $O\left(r\frac{W_i}{L} + \frac{W_i}{rL}\right)$

(using the trivial bound of h for horizontal movement)

↳ total cost of this type in the actual stage is

$$O\left(\frac{n r W_i}{L} + \frac{n V_i}{r \alpha}\right)$$

↳ $L = n^{\frac{4}{9}}$, $\alpha = n^{\frac{2}{3}}$, $r = n^{-\frac{1}{9}}$ } exercise

$$O\left(W_i n^{\frac{4}{9}} + V_i n^{\frac{4}{9}}\right)$$

$$\left(\in O\left(W_i n^{\frac{4}{9}} \log n\right)\right)$$

3. We now consider the small sweeps that lead to inactive stripes.

Problem here: may not advance $\frac{\alpha}{q_j}$ in x-direction

↳ we will use the fact that each stripe is marked inactive at most once per phase

↳ Number of stripes marked inactive in subphase j is at most $a_j - a_{j+1}$.

Therefore in subphase j the cost of this is bounded by:

$$O\left((a_j - a_{j+1}) \left[\overbrace{\frac{\alpha}{a_j} \frac{rW_i}{L} + \frac{\sqrt{L}}{a_j r} \frac{W_i}{\sqrt{L}}}^{BLA} \right] \right)$$

If the phase ends after S subphases, the portion of the cost in the phase is

$$O\left(\sum_{j=1}^{S} (a_j - a_{j+1}) \cdot BLA \right)$$

which is $O\left(W_i \left(\frac{\alpha r}{L} + \frac{1}{r} \right) \sum_{j=1}^{S} \frac{a_j - a_{j+1}}{a_j} \right)$

↳ (EXERCISE)

Since $a_1 \leq 2\sqrt{L} + 1$

and there are at most

$O(\sqrt{L})$ phases in a stage,

the total cost is

$$O\left(\sqrt{\Delta}\, w_i \log L \cdot \left(\frac{\alpha r}{L} + \frac{1}{r}\right)\right)$$

Which is in

$$O\left(w_i\, n^{\frac{4}{3}} \log n\right)$$

$$\in O\left(w_i\, n^{\frac{4}{3}} \log n\right)$$