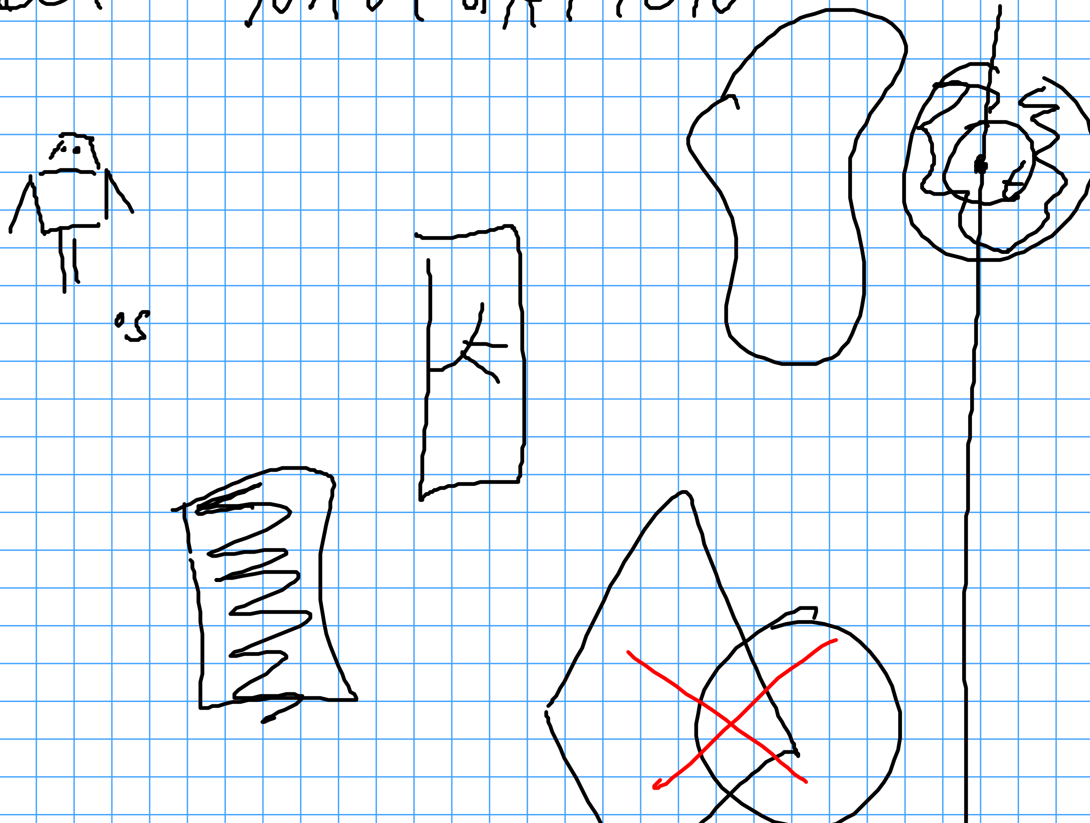
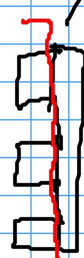
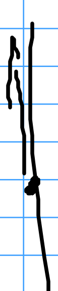
# ROBOT NAVIGATION




- obstacles are opaque, impenetrable, non overlapping
- none of the obstacles blocks or encloses s or t
- inside the unit square in each obstacle is possible
- if two obstacles touch, the robot can squeeze between them





- robot has no vision, objects can only be explored by bumping into them

⇒ GOAL: minimize distance to reach the target

1. Non-convex Obstacles

↳ construct a maze


$O(pd)$

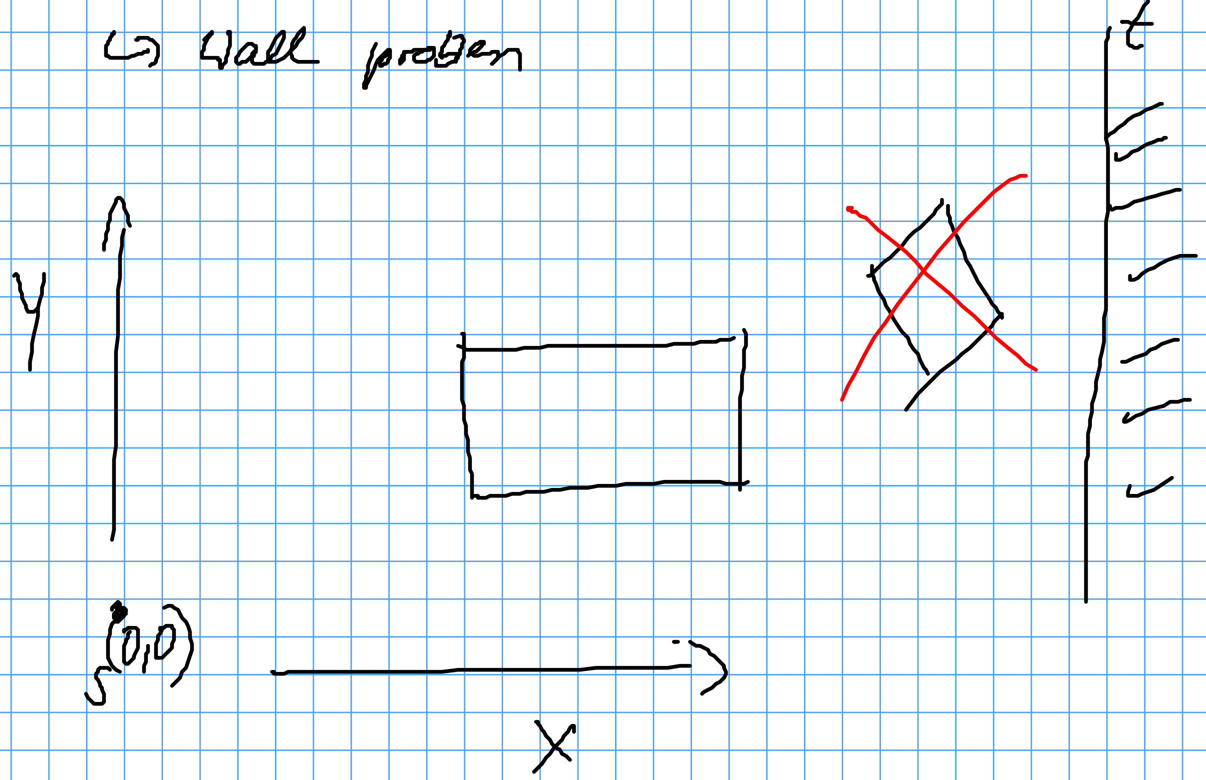
↳ robot has to walk a distance of $2d$ to explore a corridor

↳ if evil person determines opening : total walk distance in $O(pd)$

$n$-optimal path ⇒ no $\Omega(n)$ guarantee possible

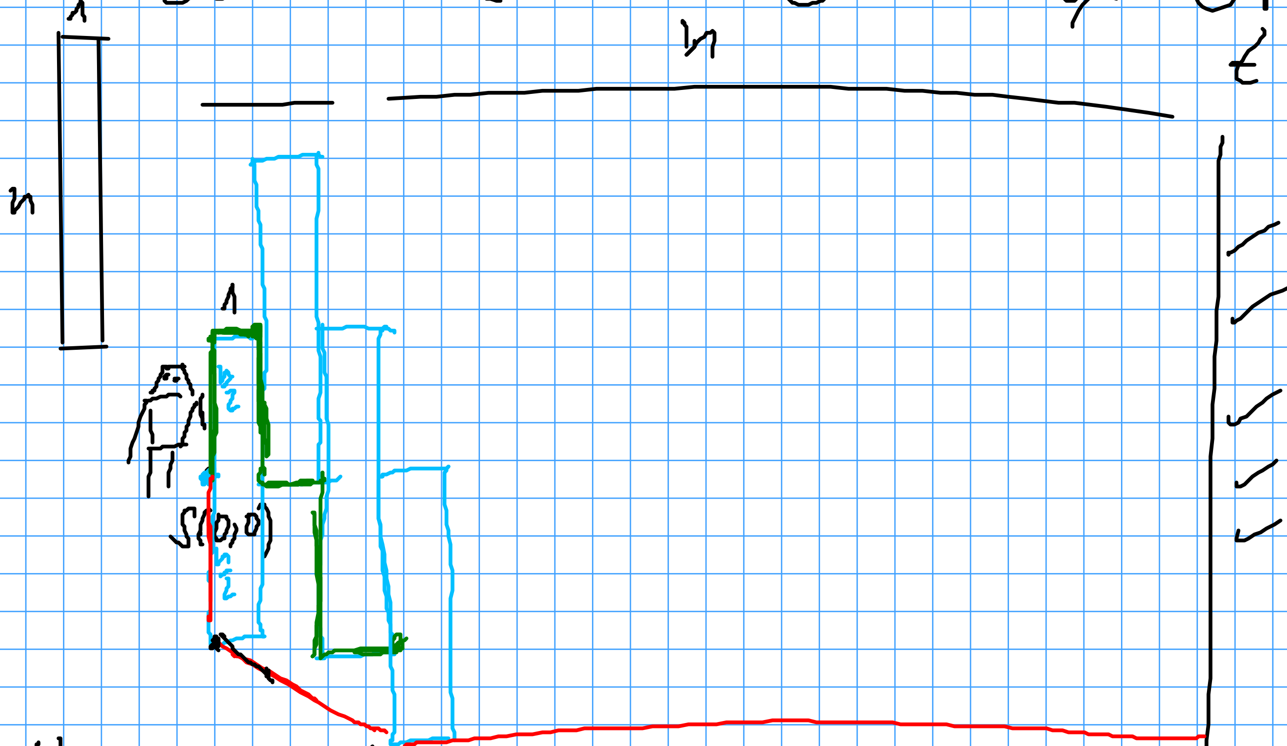
Convex Obstacles

↪ Wall problem




obstacles are axes-parallel rectangles

• Deterministic Lower Bound of $O(\sqrt{n})$




Evil person throws obst. in the way of the robot to elongate its path as much as possible.

Strategy: If the robot reaches a new X-coordinate that allows for placing a new (non-overlapping) obstacle, the evil person places the obstacles such the robot is in the middle of the obstacle's left side

Theorem: The path the robot takes is at least a factor $O(\sqrt{n})$ longer than the optimal path.

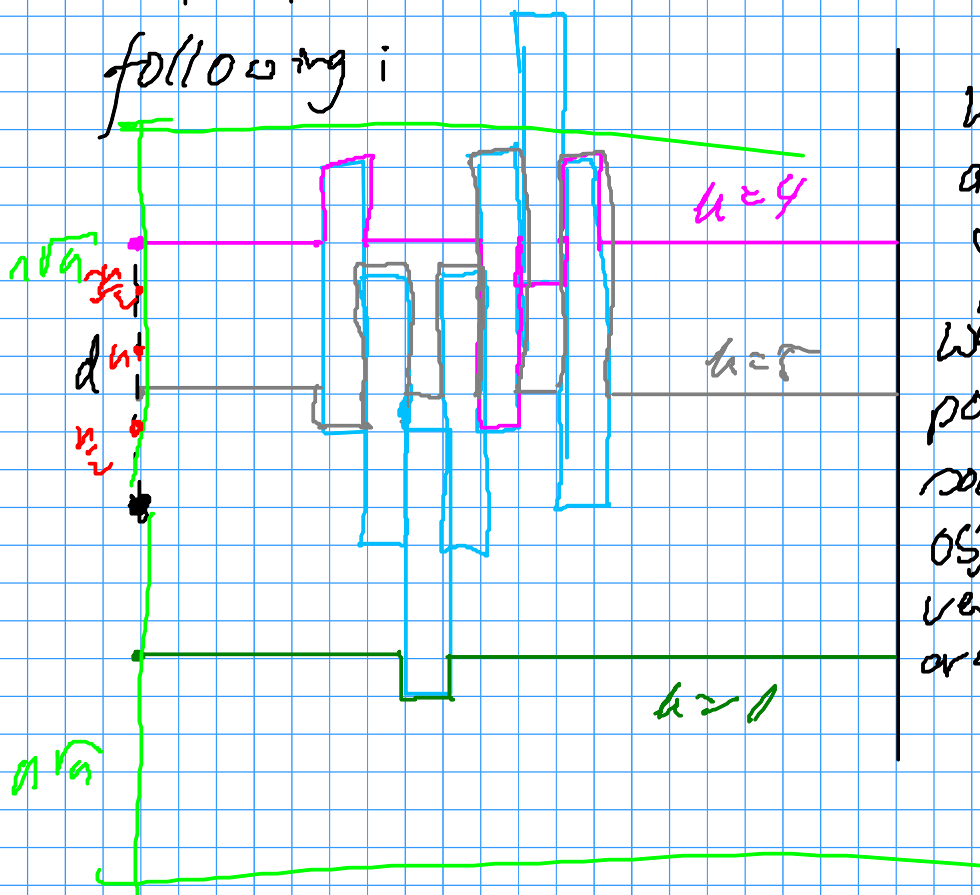
Proof: First we construct lower bound for the path of the robot.

- For each obstacle, the robot needs to walk at least $\frac{n}{2}$ in vertical direction to circumvent it. Because the distance of $s$ to $t$ is $n$ and every obstacle has a width of 1, the evil person places exactly $n$ obstacles.
- distance the robot travels is

$$\Theta(n^2)$$

If we can prove the existence of a path from $s$ to $t$ of length $O(n\sqrt{n})$, the theorem follows.

A $y$-path is defined as the following:




hold to a fixed $y$-coordinate whenever its possible, as soon as hitting object, circumvent in an arbitrary way

length of a $y$-path that intersects $k$ obstacles is bounded by:

$$d + 2kn + n$$

→ costs in horizontal direction

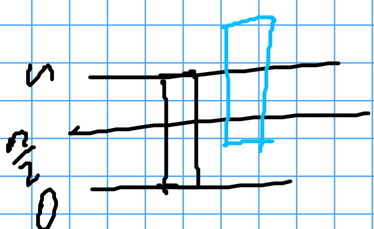
initial costs to get to $y$

→ each circumvention costs at most $2n$ in vertical direction

Want to show: can achieve that $d \in O(n\sqrt{n})$ and $a \in O(\sqrt{n})$ at the same time

For y-coordinates we consider the range $[-n\sqrt{n}, n\sqrt{n}]$.

How many y-coord. in this range are multiples of $\frac{n}{2}$?




$2n\sqrt{n}$ total interval length

↳ at most $2\sqrt{n}+1$ multiples of $\frac{n}{2}$ in that

$\Longrightarrow$ There exists a y-coordinate for which the respective y-path intersects at most $O(\sqrt{n})$ many obstacles.

Otherwise # obstacles $> \underbrace{(2\sqrt{n}+1)\,O(\sqrt{n})}_{O(n)}$

$\not\leq$ n obstacles

Hence there is some coordinate which assures that $d < \sqrt{n}\,n$ and the

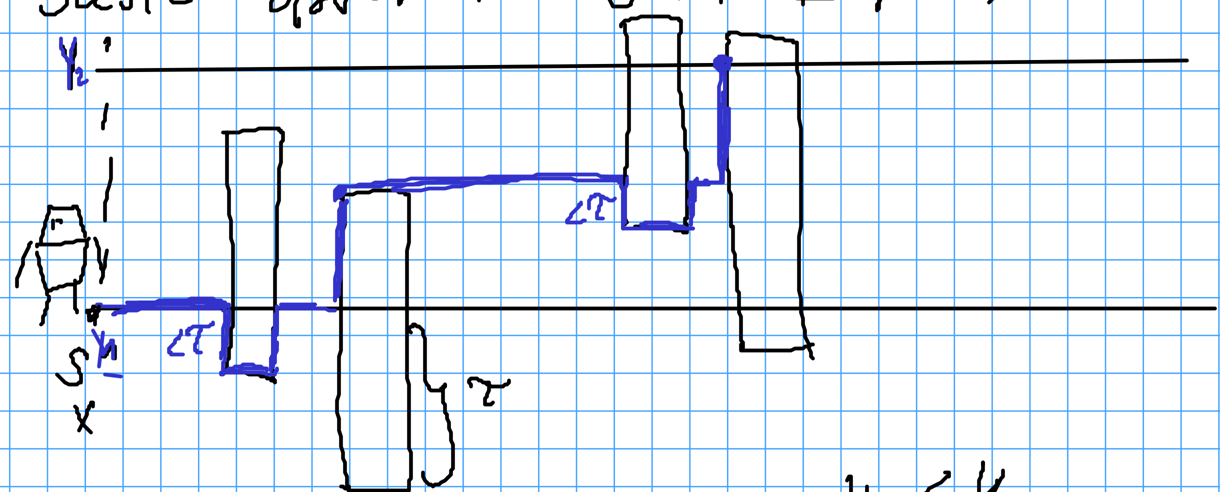
Vertical path distance around the obstacles is $O(k n) \in O(n\sqrt{n})$

$\Rightarrow$ total path length $O(n\sqrt{n})$ //

• A $O(\sqrt{n})$ - competitive Strategy

↳ matches the lower bound, so its asympt. optimal

Basic operation: SWEEP, $\tau$




$y_1 < y_2$

UPWARDSSWEEP$(x, y_1, y_2, \tau)$

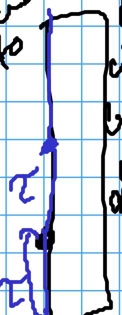
$p \leftarrow (x, y_1)$

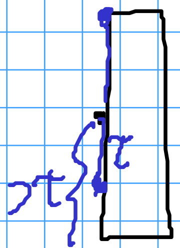
**while** actual y-coord. is $< y_2$ **do**

• move right until bumping into an obstacle O

**if** (one corner of O is less than $\tau$ away) **do**

blind robot: check function circular to $\tau$ equal cost with known distance






- circumvent O via this corner
- return to previous $y$-coordinate

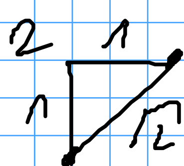
else

move to upper left corner

(DOWNWARDS SWEEP: $y_1 > y_2$, go to lower corner if $\tau$-circumvention is not possible)

Observation: An optimal path fulfilling the following properties is only a factor of $\sqrt{2}$ longer than any optimal path:

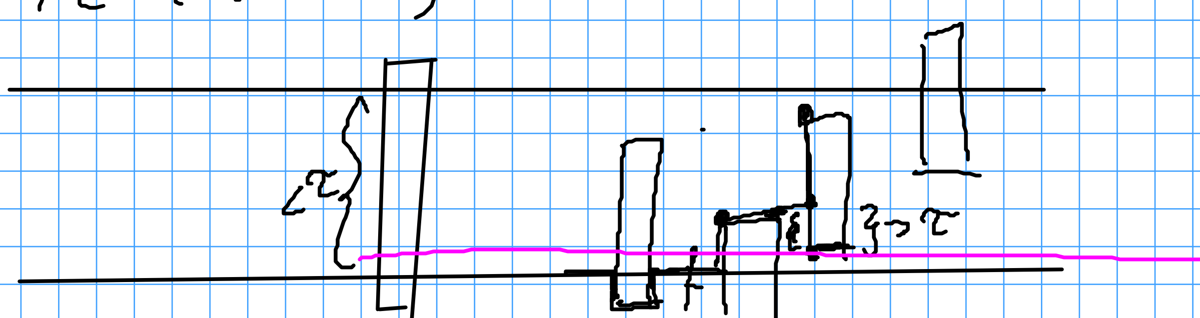
Manhattan path
- only horizontal/vertical moves are allowed
- $x$-coordinate never decreases




We are going to show, that a path determined by alternating upwards/downwards sweeps in a suitable corridor is no longer than $O(\sqrt{n})$ optimal Manhattan path length (and so no longer than $O(\sqrt{n})$ the optimal path length according to the observation).

Lemma: Let a phase be an upwards sweep followed by a downwards sweep or vice versa with parameter $\tau$. Let $Q$ be a Manhattan path whose $x$-coordinate never decreases. The total length of vertical parts of $Q$ to go from the lower to the upper $y$-coordinate and back is at least $\tau$.

(If no obstacle is larger than the corridor)



Consider the obstacle that caused an upwards move during the upwards sweep, and an obst. that causes a downwards in the downwards sweep.

Any y-coordinate in the corridor, intersects at least one obstacle (otherwise walking horizontally to the target is possible).

Between the x-coordinates where the robot starts and ends the bottom left corner of an obstacle must be lower than the top right corner the previous obstacle by more than $\tau$. Q must circumvent this obstacle as well $\rightarrow$ vertical path is longer than $\tau$

Let W is the total sum of all vertical movements in the optimal solution. (Manhattan-optimal)

↳ sweeps inside a corridor $[-W', W']$ with $W' \geq W$ and $\tau = \frac{W'}{\sqrt{n}}$

The choice of the corridor ensures that the optimal Manhattan path is completely contained in the corridor.

Length of the path resulting from alternating sweeps:

- first: bound on vertical parts resulting from $\tau$ - circumventions
  - ↳ to get to the circumvention case: at most $3\tau = 3\frac{W'}{\sqrt{n}}$ $+\tau$ to connect
  - ↳ at most $n$ of those

$$\Rightarrow \frac{4W' \cdot n}{\sqrt{n}} \in O(W'\sqrt{n})$$

↳ vertical distance made in upward / downward moves.

Vertical distance in one phase is exactly $2W'$.

↳ We know that the vertical parts of an optimal path are $\leq W$.

↳ Total length of vertical

parts of an optimal path within a phase is at least $\tau = \frac{W'}{\sqrt{n}}$.

$\Rightarrow$ After no more than $\left\lceil \frac{W}{\tau} \right\rceil$ phases, we must reach the target wall. (because path is completely contained in the corridor)

$\hookrightarrow$ total vertical distance:

$$2W' \cdot 2 \cdot \left\lceil \frac{W}{\tau} \right\rceil \qquad \frac{W}{\frac{W'}{\sqrt{n}}} = \frac{\sqrt{n}W}{W'}$$

if $W' \leq 2W$

$$\Rightarrow 4 W' \left\lceil \frac{W}{\tau} \right\rceil \leq 8W \cdot \left\lceil \frac{W}{\tau} \right\rceil \to O(\sqrt{n})$$

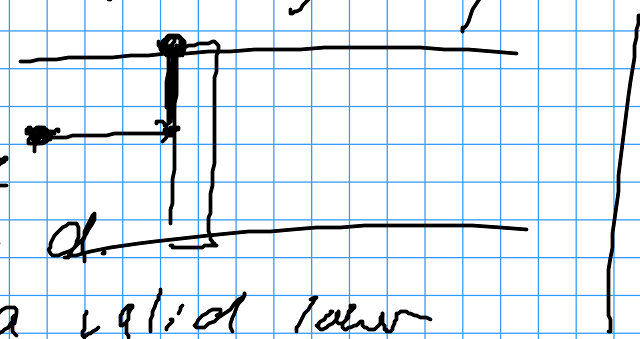
$\hookrightarrow O(W\sqrt{n})$

$\Rightarrow$ Proves that the strategy for known $W$ is $O(\sqrt{n})$-competitive.

But we do not know $W$ a priori.

$\longrightarrow$ DOUBLING

1. Move right until reaching the first obstacle.



Calculate distance to nearest corner $d$.

$\hookrightarrow$ $d$ is a valid lower bound for $W$.

(using row-path alg. as costs $w \leq 9d$)

2. For every $i > 0$ (before reaching the wall)

 - We run previous alg. with $W = d$. Stop after $\sqrt{n}+1$ phases or when reaching the wall
 - $d \leftarrow 2d$

Let $\hat{j}$ be the integer for which $2^{\hat{j}} d \leq W < 2^{\hat{j}+1} d$. During the $i^{th}$ iteration the total vertical path is in $\mathcal{O}(\sqrt{n}\, W_i)$.

When running the alg. with $w_{j+1}$ we reach the wall for sure.

$$\sum_{i=1}^{j+1} O(\sqrt{n}\, w_i) = \sum_{i=1}^{j+1} O(\sqrt{n}\, d \cdot 2^i)$$

$$= O(\sqrt{n}\, d) \sum_{i=1}^{j+1} 2^i = O(\sqrt{n}\, \underbrace{d\, 2^j}_{w})$$

$$\in O(\sqrt{n}\, w)$$

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