Information Retrieval

WS 2013 / 2014

Lecture 3, Tuesday November 5th, 2013
(Efficient List Intersection)

Prof. Dr. Hannah Bast
Chair of Algorithms and Data Structures
Department of Computer Science
University of Freiburg

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Overview of this lecture

- Organizational
  - Your experiences with Exercise Sheet 2 (Ranking)
  - Some general implementation advice
- List intersection
  - Time-measurement How-To
  - Non-algorithmic improvements
  - Algorithmic improvements: galloping-search intersect
  - Lower bound
  - **Exercise Sheet 3:** implement the galloping-search list intersection and compare to the linear-time one

Experiences with ES2 (ranking)

- Summary / excerpts last checked November 5, 15:00
  - Good exercise to understand ranking + how it works
  - Some Java folks with very long indexing times, reasons:

    Insufficient heap space ... increase with java -Xmx=3g ...

    Search doc id with ArrayList.contains ... oh oh, see Slide 5
  - In general, many programming issues, which have nothing to do with the topic of this course

    Nevertheless: important that you are learning this now!
  - Use formula formatting in PPT ... I don't like PPT formulas
  - Put the annotated slides online after the lecture ... I did !
  - Happy about extensive feedback from tutor

Your results for ES2 (ranking)

- Some interesting observations you made:
  - Time-consuming to find a good query

    But also quite instructive wrt how the ranking works
  - BM25 has a strong preference for shorter documents containing the query words

    This was not such a great advantage here, because all the documents in people.tsv were relatively short
  - In Wikipedia, document length is actually a sign of significance of an article = in our case, the person the article is about

    True! Popularity is another important ingredient of a ranking function, which we did not (yet) talk about

Using functions from a Library

- Yes, you can do that, **but**

  … **you should really know what you are doing!**

  – Example 1: using ArrayList.contains to check if an element is already in the array

  If done for each element added, takes quadratic time !

  For this application, it suffices to look at last element

  – Example 2: using std::set_intersection to implement the linear-time intersect

  Ok, if you convinced yourself that the method is doing the right thing with the right time complexity (cf. above)

  In any case, add a comment that you understood this !

Time measurement 1/3

- In **Java**
  - For **milli**second resolution

```
long start = System.currentTimeMillis();
// whatever code you want to time
long end = System.currentTimeMillis();
long millis = end – start;
```

  - For **micro**second resolution (or maybe better)

```
long start = System.nanoTime();
// whatever code you want to time
long end = System.nanoTime();
long micros = (end – start) / 1000;
```

Time measurement 2/3

- In **C++**
  - For **milli**second resolution

```
include <time.h>
clock_t start = clock();
// whatever code you want to time
clock_t end = clock();
size_t millis = 1000 * (end – start) / CLOCKS_PER_SEC;
```

  - For **micro**second resolution

```
include <sys/time.h>
struct timeval tv;
struct timezone tz;
gettimeofday(&tv, &tz);
size_t startMicros = 1000000L * tv.tv_sec + tv.tv_usec;
…
```

Time measurement 3/3

- Never rely on a single measurements
  - There can be significant variation, for example due to:

    Other jobs running on your machine + JIT compilation

    The Java garbage collector running unpredictably

    Data read from disk in the memory cache or not

    Data read from memory in the L1-cache or not

    Virtual memory addresses in the TBL cache or not
  - For ES3, **repeat** each run **10 times** and take the average

    Note: for small inputs, this can distort the actual truth, because of caching effects. But not a big issue for ES3.

Non-algorithmic improvements 1/3

- Type of array to store the inverted lists
  - **Java:** ArrayList is much worse than native [] array

    Note: elements of an ArrayList cannot be basic data types (e.g. int), but have to be objects (e.g. Integer)
  - **C++:** std::vector is as good as [] with option –O3

    Note: elements of an std::vector can be basic data types as well as objects, unlike in Java

Non-algorithmic improvements 2/3

- Branch prediction
  - This pertains to all conditional parts in your code, in particular, if – then – else parts
  - Modern processors do pipelining = speculative execution of future instructions before the current ones are done
  - For conditional parts they have to guess the outcome
  - So good to **minimize** amount of conditional parts

Non-algorithmic improvements 3/3

- Simple code in loops
  - Within a loop try to keep the number of variables small

    This will allow the compiler to use (fast) registers

    Don't worry about **constants** though, modern compilers figure out that they don't need a variable for those
  - In C++, when you call a function that does something very simple very often, then **inline** it

    Inline = put the code in the header file + precede by the keyword **inline** (the latter is not necessary for short code)

    The compiler will then avoid the function call and instead put a copy of the code of the fct. at each place you call it

Algorithmic improvements 1/4

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- Binary search in the longer list
  - Call the smaller list A, and the longer list B
  - Search each element from A in B, using binary search

    k = #elements in A, n = #elements in B
  - This has time complexity $\Theta(k \cdot \log n)$

$\Theta(\log n)$ time per element of A

13 15 17

A

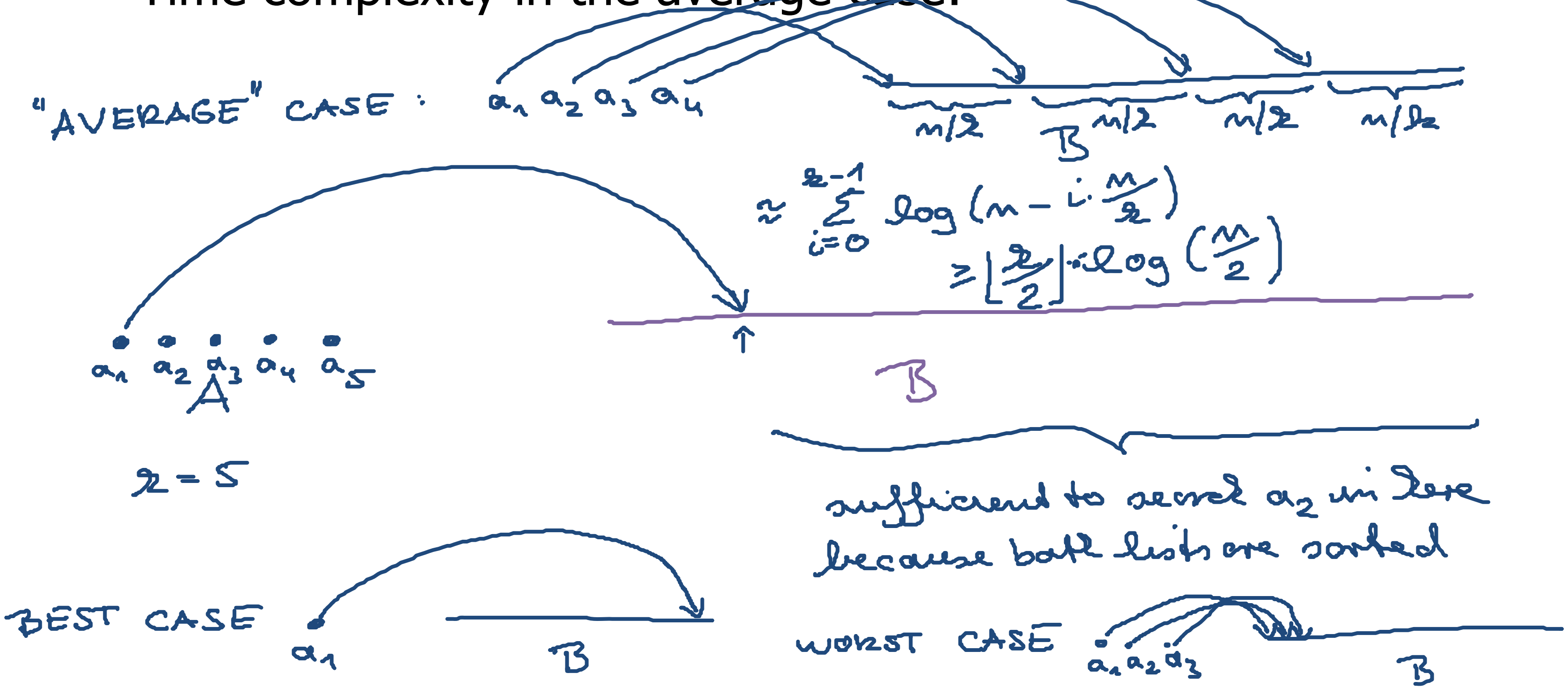
3 5 9 10 11 14 21 27 43

B

NOTE: for $k = n$, this is $\Theta(n \cdot \log n)$, which is worse than the $\Theta(n)$ of the linear-time intersect.

Algorithmic improvements 2/4

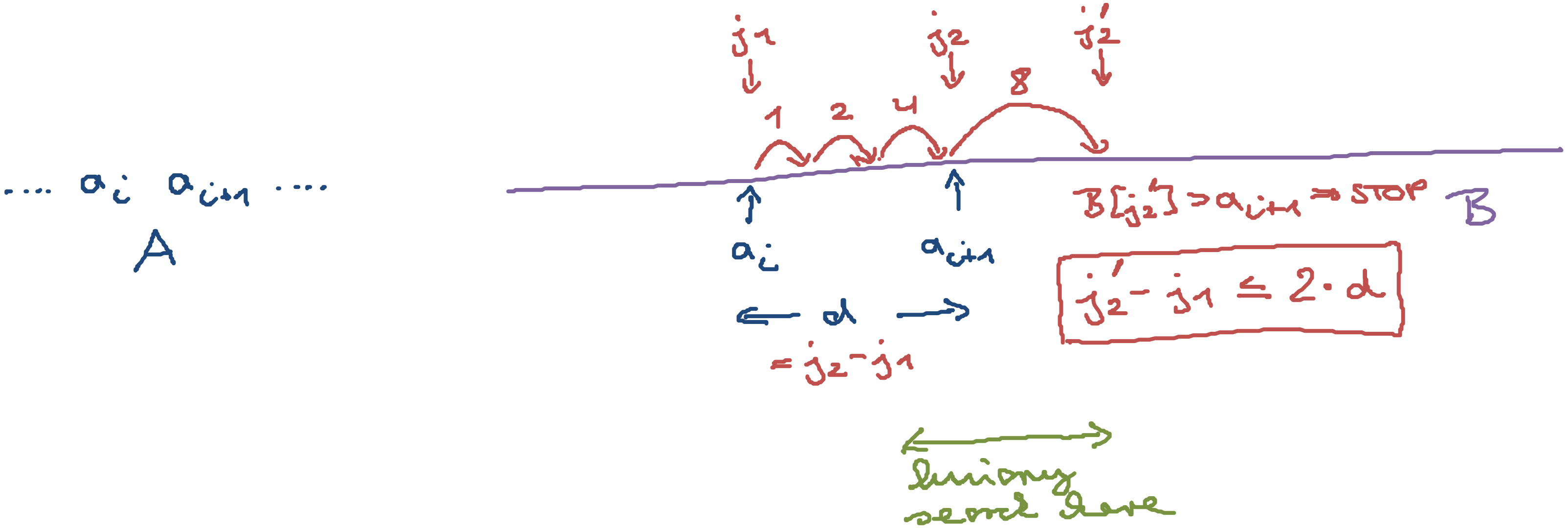
- Binary search only in the remaining part of B
  - Time complexity in the best case: $O(k + \log n)$
  - Time complexity in the worst case: $O(k \cdot \log n)$
  - Time complexity in the average case: $\Omega(k \cdot \log n)$



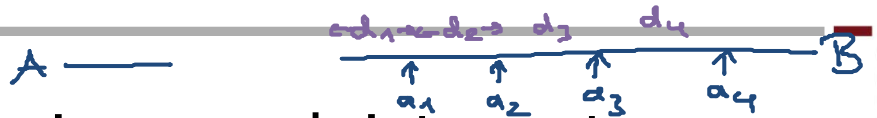
Algorithmic improvements 3/4

- "Galloping" search in B
  - **Goal:** when elements A[i] and A[i+1] are located at positions j_1 and j_2 in B, then, with $d := j_2 - j_1$ ("gap"):

    spend only time **Θ(log d)** to locate element A[i+1]
  - **Idea:** first do an exponential search, to get an upper bound on the range, then a binary search as before



Algorithmic improvements 4/4



- Time complexity of galloping-search intersect
  - Let $d_1, \ldots, d_k$ be the gaps between the locations of the k elements of A in B

    d_1 = from beginning to first location
  - Note that $\Sigma_i\, d_i \leq n$ = the number of elements in B
  - Then the time complexity is $O(\Sigma_i \log d_i)$
  - **Goal:** find a formula that is independent of the d_i
  - **Idea:** maximize $\Sigma_i \log d_i$ under the constraint $\Sigma_i\, d_i \leq n$
  - This is called optimization with side constraints or Lagrangian optimization

    shown by example on the next slide …

Lagrangian Optimization

- Maximize $\Sigma_i \log d_i$ under the constraint $\Sigma_i d_i \leq n$

Let's max $\Sigma_i \ln d_i$, note $\ln \sim \log$

$$\mathcal{L} := \sum_{i=1}^{k} \ln d_i + \lambda \cdot \left(n - \sum_{i=1}^{k} d_i\right)$$

$$\frac{\partial \mathcal{L}}{\partial d_i} = \frac{1}{d_i} - \lambda \overset{!}{=} 0 \implies \lambda = \frac{1}{d_i} \implies \text{all } d_i \text{ equal}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = n - \sum_{i=1}^{k} d_i = 0 \implies \sum_{i=1}^{k} d_i = n \implies d_i = n/k$$

So, we have a local extremum at $d_i = n/k$

$$\frac{\partial^2 \mathcal{L}}{\partial d_i^2} = -\frac{1}{d_i^2} < 0 \implies \text{MAX.}$$

$$\implies \sum_{i=1}^{k} \log d_i \leq k \cdot \log\left(\frac{n}{k}\right)$$

+ also check border cases: one $d_i = n$, all others 0

Skip pointers

- A heuristic approach
  - **Idea:** place skip pointers at "strategic" places in B, to potentially enable skipping large parts of B

    The heuristic part is to decide where these "strategic" places are + how much to skip … see references for details
  - **Advantage:** very simple to implement, in particular, simpler than galloping search

    For ES2, you will implement the galloping search though

Lower bound 1/2

- Let's summarize our upper bounds so far
  - As before, let k = the size of the smaller list, and let n = the size of the larger list
  - Linear-time intersection: $O(k + n)$
  - Galloping-search intersection: $O(k \cdot \log (n/k))$

Lower bound 2/2

3 5 7 (A) 105 107 . . . . . (B)

- Can we do even better than $k \cdot \log(n/k)$?
  - No, at least not for general inputs

    For particular inputs, you can finish in time $O(1)$ … why?
  - Recall the lower bound for comparison-based sorting:

    There are $n!$ possible outputs, we have to differentiate between all of them, and only two choices per step

    Hence #steps required $\geq \log_2(n!) = \Omega(n \cdot \log n)$
  - We can use a similar argument for intersection / union:

    There are n+k over k ways how the k elements from A can be placed within the n elements from B, …

    Hence #steps required $\geq \log_2(n/k)^k = k \cdot \log_2(n/k)$

References

- In the Raghavan/Manning/Schütze textbook

  Section 2.3: Faster intersection with skip pointers

- Relevant Papers

  A simple algorithm for merging two linearly ordered sets

  F.K. Hwang and S. Lin SICOMP 1(1):31–39, 1980

  A fast set intersection algorithm for sorted sequences

  R. Baeza-Yates CPM, LNCS 3109, 31–39, 2004

- Relevant Wikipedia articles

  http://en.wikipedia.org/wiki/Lagrange_multiplier