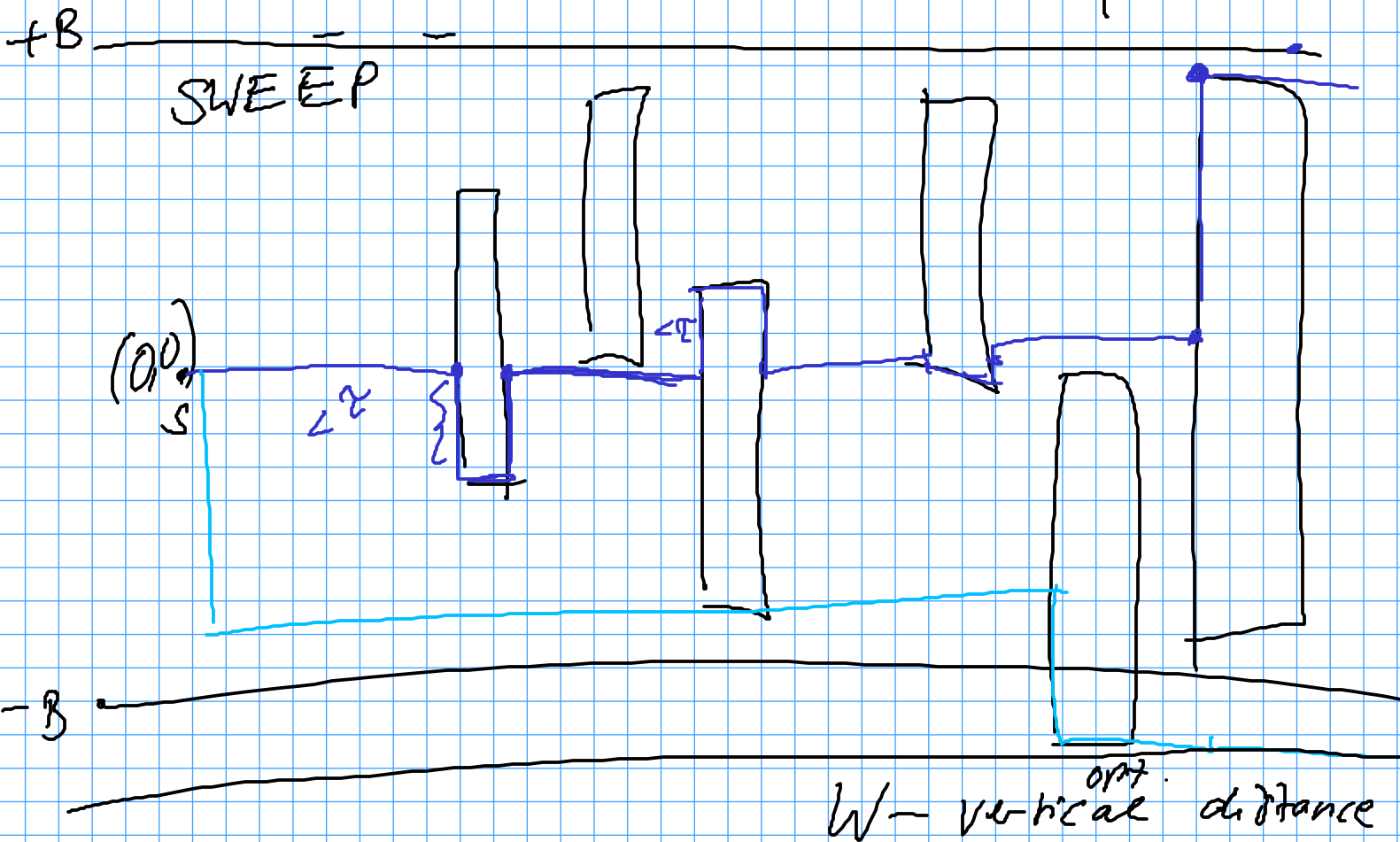
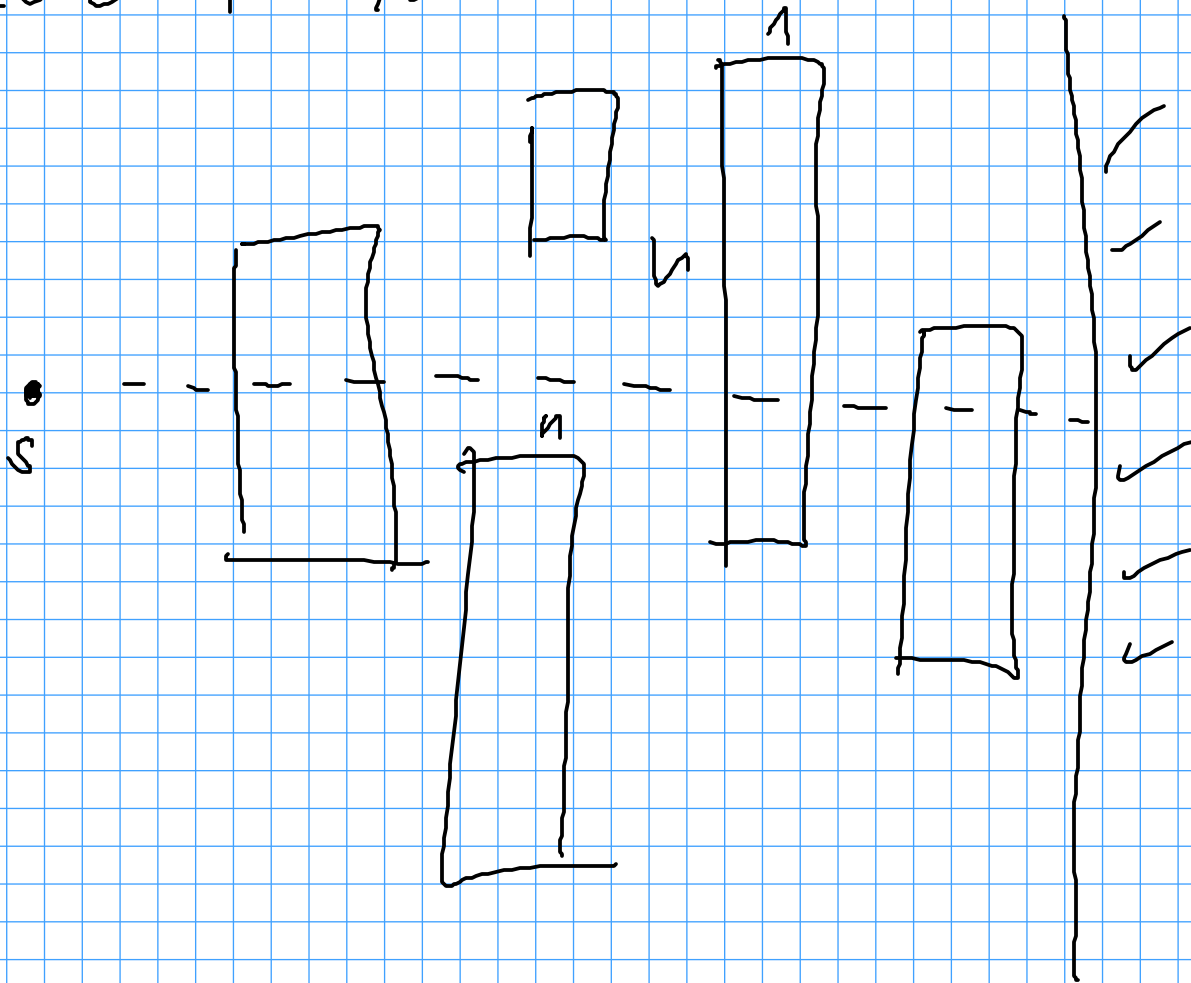
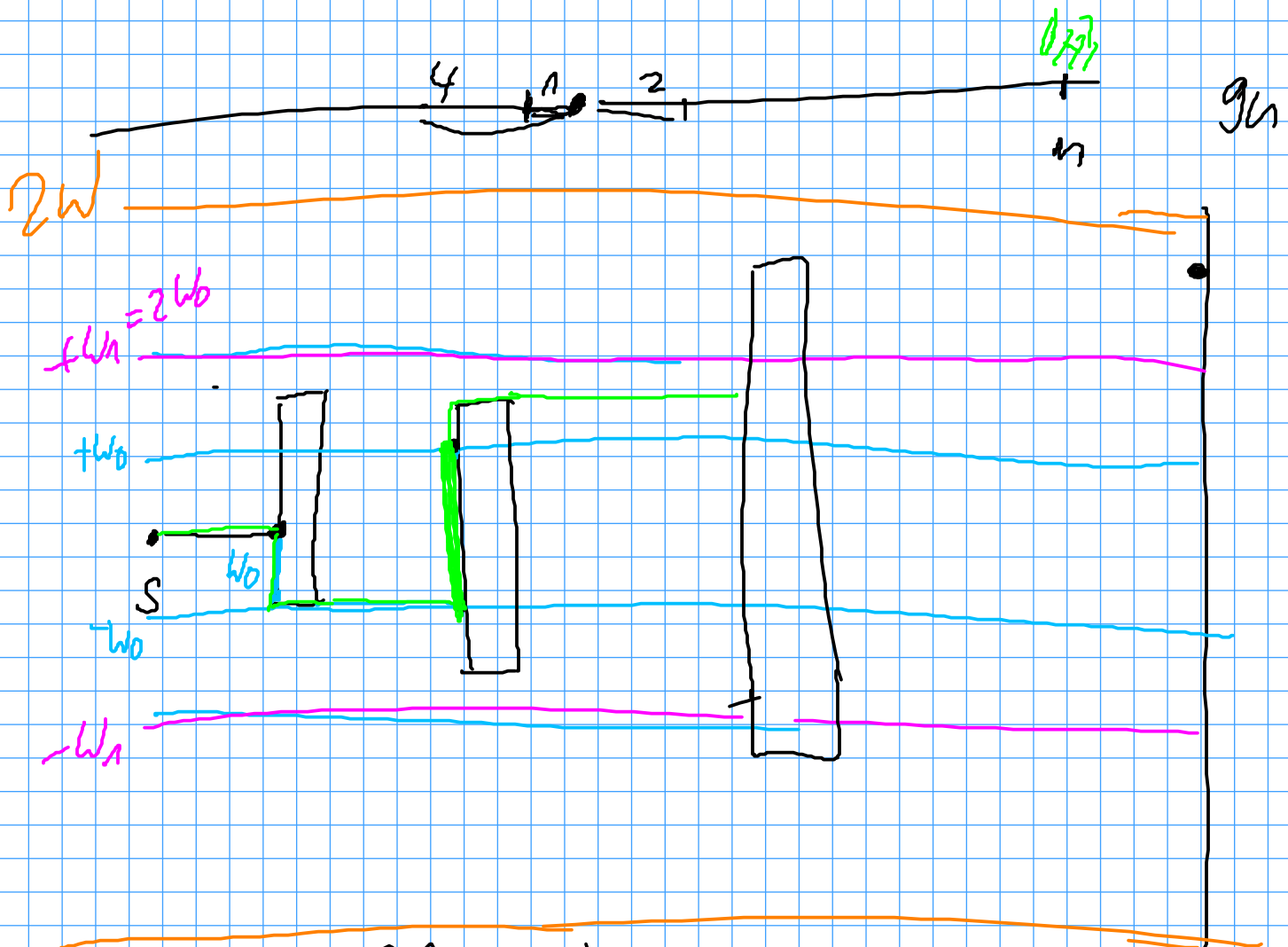


# ROBOT NAVIGATION





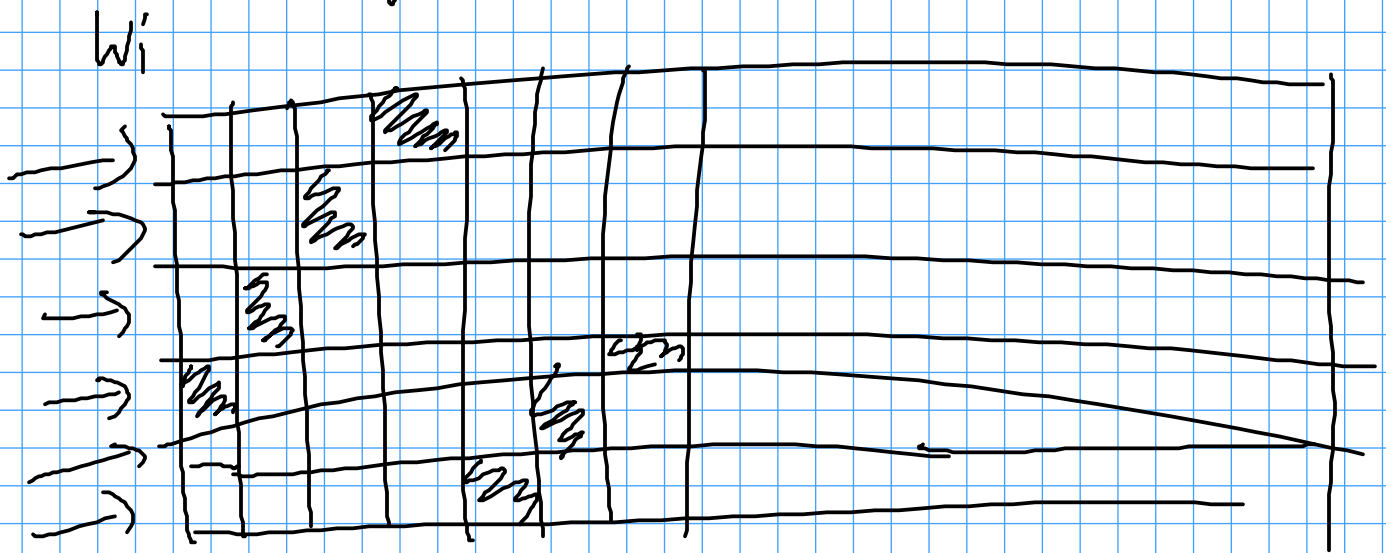
$$O(W \sqrt{n})$$

$O(\sqrt{n})$  - competitive

,  $\sqrt{n}$  - lower bound

Randomized Approach:  $O(n^{\frac{4}{9}} \log n)$   
competitive factor

Stage  $i$



We will describe an algorithm with the following properties:

1. The cost in stage  $i$  is  $O(W_i n^{\frac{4}{5}} \log n)$

2. For  $n$  large enough, for each  $i$  with  $W_i > W$ , the prob. that the robot reaches the wall is  $\geq \frac{3}{4}$ .

Theorem

This alg. is  $O(n^{\frac{4}{5}} \log n)$   
competitive

Proof: Let  $j$  be the integer such that  $2^j \cdot w_0 \leq W < 2^{j+1} w_0$ .

The expected cost of the strategy is

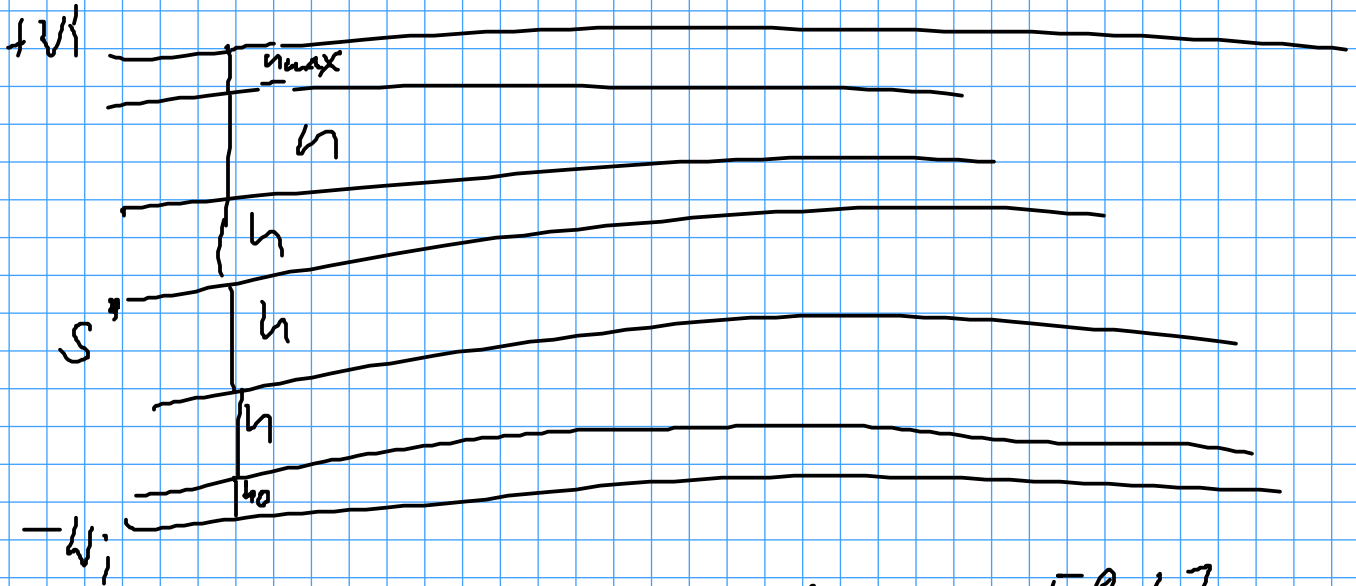
$$\begin{aligned}
 E(X) &= \left( \sum_{i=1}^{j-1} w_i n^{\frac{4}{9}} \log n + \sum_{i=j}^{\infty} \left(\frac{1}{4}\right)^{i-j} w_i n^{\frac{4}{9}} \log n \right) \\
 &= \left( n^{\frac{4}{9}} \log n \left( \underbrace{\sum_{i=1}^{j-1} 2^i w_0}_{O(2^j w_0) = O(W)} + \sum_{i=j}^{\infty} \left(\frac{1}{4}\right)^{i-j} 2^i w_0 \right) \right) \\
 &= O\left( n^{\frac{4}{9}} \log n \cdot (W + W) \right) \leq 2W n^{\frac{4}{9}} \log n \\
 &= O(W n^{\frac{4}{9}} \log n)
 \end{aligned}$$

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Details of a Stage

- subdivide corridor in stage  $i$

$([-W_i, +W_i])$  in stripes of height  $h$ , with  $h = \frac{W_i}{n^{\frac{2}{3}}} = \frac{W_i}{\sqrt{L}}$



- choose random value in  $[0, h]$ , call it  $h_0$
- corridor stripes are then

$$[-W_i, -W_i + h_0]$$

$$[W_i + h_0, -W_i + h_0 + h]$$

⋮

last corridor is bounded by  $+W_i$

$$\hookrightarrow \# \text{ stripes: } 2 \left\lceil \frac{W_i}{h} \right\rceil + 1$$

stripe labels

- active
- inactive
- useless

Phase: all stripes are active,  
 perform subplans until target  
 wall is reached or all  
 stripes are non-active  
 (in active or waiting)

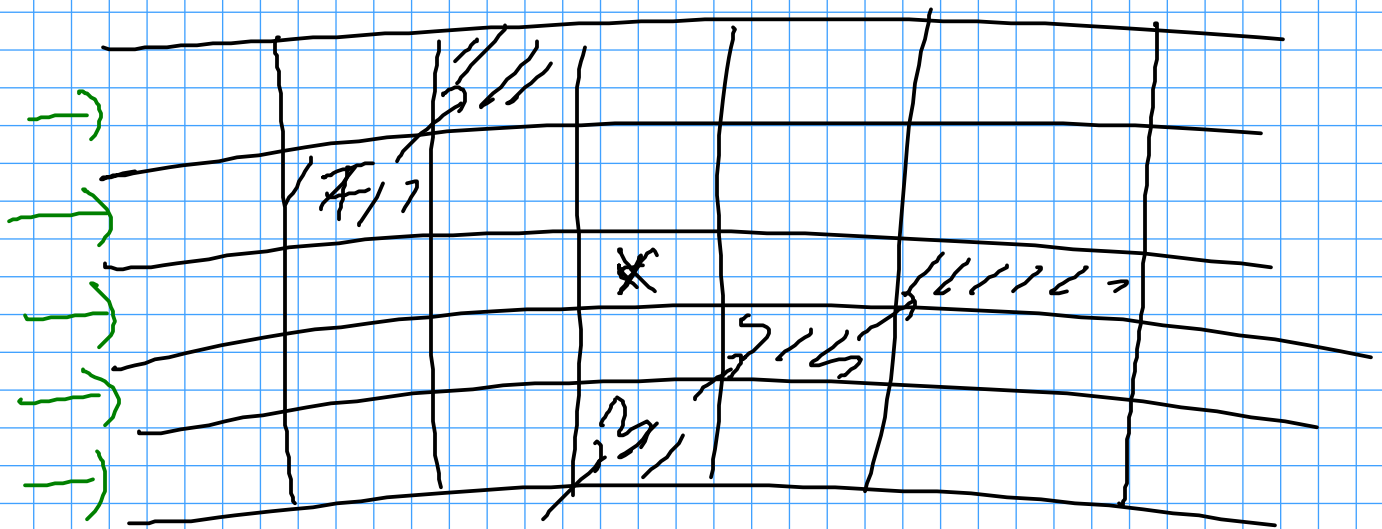
(1) once all of the stripes  
 are non-active after perfor-  
 ming sufficiently many subplans,  
 we start a new phase

$$\alpha = n^{\frac{2}{j}}$$

Subphase:

$\alpha$

active



# of active stripes at the beginning  
 of subphase  $j$  is  $a_j$

(1) divide the stripes into boxes of  
 equal width  $\frac{\alpha}{a_j}$

• choose one active stripe u.a.r., then

construct a 'desired' diagonal.



Go always to the upper left field, until there is no more stripe left above, then use a wrap-around to start at the bottom and proceed until every stripe is covered.

- How should the robot find its way to the next desired field?

Three cases

(1) robot is in the desired field

(2) inside the stripe containing the field but left of it

(3) attempting to move to the correct stripe



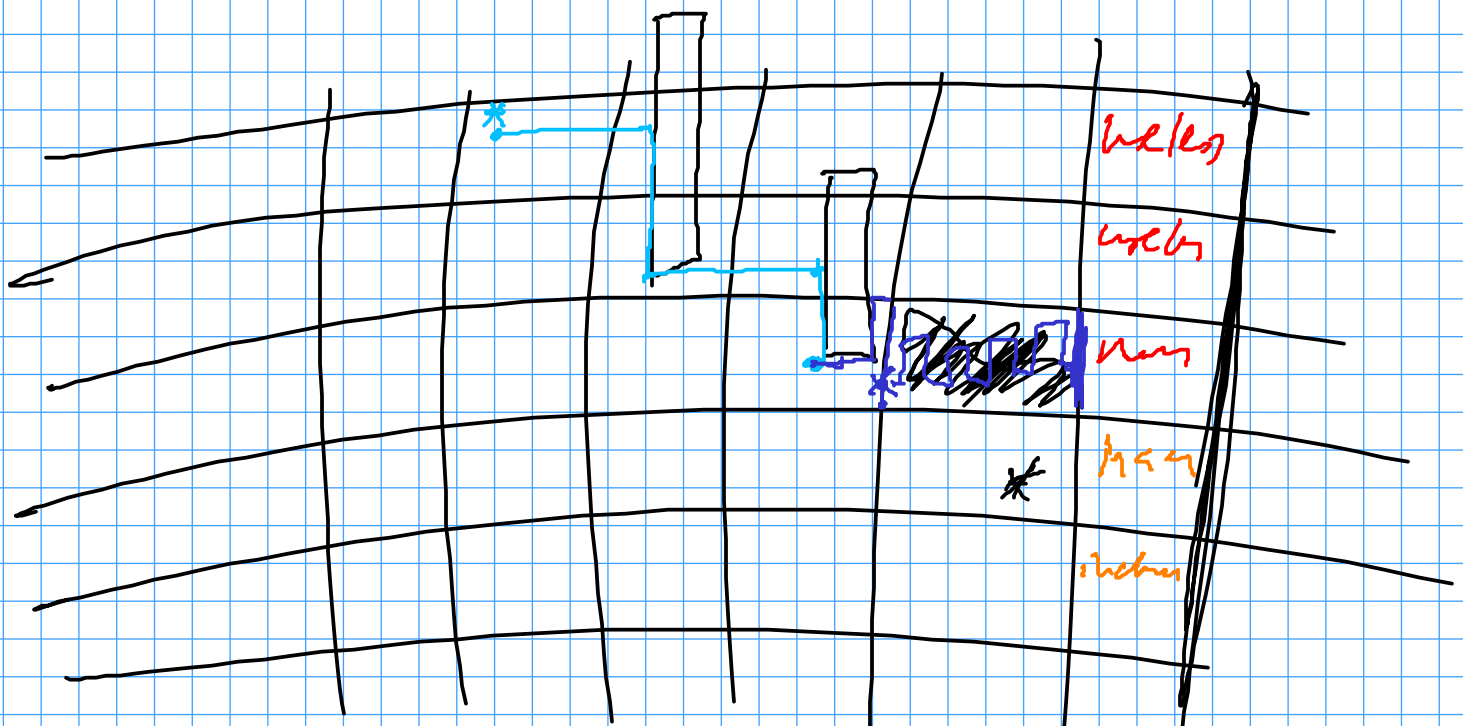
(1) go greedily to the bottom of the field  $\rightarrow$  perform sweeps with

$$r_n = r \cdot \frac{w_i}{L}$$
$$r = n \frac{1}{5}$$
$$L = n \frac{4}{5}$$

Small sweeps

(2) move towards the field  
by using sweeps with  
parameter  $\tau_2 = \frac{W_i}{L}$  bsg  
sweeps

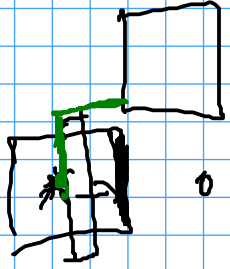
(3) go greedily towards the  
correct stripe: go right  
as long as possible, if an  
obstacle is hit, go up or down  
whatever brings you closer to  
the correct stripe



If it happens that the robot is  
X-coordinated because the right edge



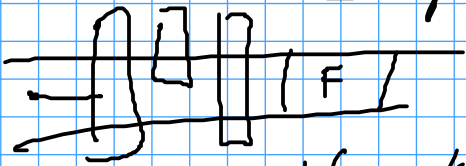
of the desired field before its stripe is reached, then update the desired field to the next one on the 'diagonal'!



• If the robot performs in the desired field  $\left\lceil \left\lfloor \frac{a}{r} \right\rfloor \left\lfloor \frac{a}{a_j} \right\rfloor \right\rceil$  full small sweeps,

we declare the stripe **inactive**.

After this number of sweeps or after reaching the right edge of the field, we proceed to the next desired one.



• If the robot is in mode (2) then he stops after at most  $\sqrt{2} = 1.414$  many big sweeps (covering over the whole phase)

as if they are used up, declare the stripe **useless**

By the property of the sweep,  
a stripe is declared useless  
only if the cost of any path  
lying entirely in this stripe is at  
least  $\frac{W_i}{V_i} = h$ .

Final case: robot has no more  
desired fields (because whole column  
only consist of useless or inactive  
fields)

'fill-up case'

↳ still need to get to the  
right edge of the box with  
width  $\alpha$

⇒ greedily move upwards  
(with wrap around) to the  
next non-useless stripes  
and repeat. If all stripes  
are useless, then we start  
a new phase.

## Exercise

Let  $a_1 \geq a_2 \geq \dots \geq a_{e-1} > 0$   
be integers. Define  $a_e = 0$ .

$$\text{Then } \sum_{j=1}^{e-1} \frac{(a_j - a_{j+1})}{a_j} \leq 1 + \ln a_1.$$

Lemma: The cost of the alg.  
at stage  $i$  is  
 $O(w_i \cdot n^{\frac{4}{3}} \log n)$ .

Proof: We analyze the cost of the  
alg. along  $y$ -direction.  
We partition it into several  
parts

1. going to the correct field  
costs  $O(w_i)$  for each subphase.  
(Including fill-up in the end).

All subphases cover a distance  
 $\alpha$  in positive  $x$ -direction (except  
maybe the last one)

$\Leftrightarrow$  at most  $\frac{n}{\alpha}$  subphases  
of full length

$\Rightarrow$  at most  $32 \sqrt{n}$  phases in  
a stage  $\Rightarrow$  clearly in  
one stage the robot can  
cover at most distance  $n$   
along the  $x$  direction, there-  
fore the total of the costs  
over all subphases in the stage  
is  $O\left(W_i \left(\frac{n}{\alpha} + \sqrt{n}\right)\right)$

$$\alpha \geq n^{\frac{1}{3}} \quad \alpha \leq n^{\frac{4}{9}} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{exercise}$$

$$\Rightarrow O\left(W_i n^{\frac{4}{9}} \log n\right)$$

2. Now consider small sweeps inside  
fields in the case that we do  
not mark the stripe inactive

Denote by  $a_j$  the number of  
stripes active at the beginning of  
subphase  $j$ . Each field has  
 $a$  width of  $\frac{\alpha}{a_j}$ .

The cost in the field is

$$I_1 \quad O\left(\frac{\alpha}{a_j} \cdot \frac{v w_i}{L} + \sqrt{\frac{v}{a_j r}} \sqrt{\frac{w_i}{L}}\right)$$

1) first term is the cost of going around at most  $\frac{\alpha}{a_j}$  many obstacles which are hit within  $T_1$  of the least corner  
( $T_1 = \frac{v w_i}{L}$ )

2) second term is the cost of doing at most  $\left\lceil \frac{\sqrt{L}}{a_j r} \right\rceil$  full sweeps in the corridor of height  $h = \frac{w_i}{\sqrt{L}}$

3) This gives us a cost of  $O\left(r \frac{w_i}{L} + \frac{w_i}{r \alpha}\right)$

(using the initial bound of  $h$  for horizontal movement)

↳ total cost of this type in the actual stage is

$$O\left(\frac{h v w_i}{2} + \frac{n v_i}{r \alpha}\right)$$

$$\text{↳ } \alpha = h^{\frac{4}{3}}, \alpha = \frac{2}{h^{\frac{2}{3}}}, v = h^{\frac{1}{3}} \quad \left. \begin{array}{l} \text{worst} \\ \text{case} \end{array} \right\}$$

$$O\left(w_i n^{\frac{4}{3}} + v_i h^{\frac{4}{3}}\right)$$

$$\left( \in O\left(w_i n^{\frac{4}{3}} \log n\right) \right)$$

3. We now consider the small sweeps that lead to inactive stripes.

problem here: may not advance

$$\frac{\alpha}{q_j} \text{ in } x \text{ direction}$$

↳ we will use the fact that each stripe is marked inactive at most once per phase

↳ Number of stripes marked inactive in subphase  $j$  is at most  $a_j - a_{j+1}$ .

Therefore in subphase  $j$   
 the cost of this is  
 bounded by:

$$O\left((a_j - a_{j+1}) \left[ \frac{\alpha}{a_j} + \frac{rW_i}{2} + \sqrt{r} \frac{W_i}{a_j} \right] \right)$$

BLA

If the phase ends after  $S$   
 subphases, the portion of the cost  
 in the phase is

$$O\left(\sum_{j=1}^S (a_j - a_{j+1}) \cdot \text{BLA}\right)$$

which is  $O\left(W_i \left(\frac{\alpha r + 1}{2r}\right) \sum_{j=1}^S \frac{a_j - a_{j+1}}{a_j}\right)$

↳ (EXERCISE)

Since  $a_1 \in 2r + 1$

and there are at most

$O(\sqrt{r})$  phases in a stage,

the total cost is

$$O\left(\sqrt{n} \sum_{i=1}^n w_i \log 2 \cdot \left(\frac{a_i}{2} + \frac{1}{r}\right)\right)$$

Which is in

$$O\left(\sum_{i=1}^n w_i n^{\frac{4}{3}} \log n\right)$$

$$\in O\left(\sum_{i=1}^n w_i n^{\frac{4}{3}} \log n\right)$$